Unit 03

VARIATION

Q.1 Define following terms.

(i) Ratio: **(L.B 2008,2009)**

Ans: Ratio is a relation between two quantities of the same kind and having same units. The ratio tells that what part one quantity of another or how much time one quantity is of another.

The ratio between two quantities a and b is denoted as a : b and is defined as:

$$a : b = \frac{a}{b} = a \div b \quad a, b \neq 0.$$

(ii) Proportion: **(L.B 2009)**

The statement of equality between two ratios a : b and c : d is called a proportion for the four quantities a, b, c and d, in general , it is written as:

a : b :: c : d.

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

Product of extremes = Product of means

(iii) Types of variations:

(L.B 2009, 2010)

Variation are of two types.

(i) Direct Variation (ii) Inverse variation

(iv) Direct Variation:

(L.B 2009, 2010)

A relation between two quantities such that an increase in one quantity causes an increase in other quantity or a decrease in one quantity causes a decrease in the other quantity in the same ratio is called direct variation. If two quantities x and y vary directly, then

$$y \propto x$$

$$y = kx$$

$$\frac{y}{x} = k$$

Examples

(i) work and days

(ii) temperature and volume

(iii) weight and mass of a body

(iv) circumference and radius of a circle.

(v) Inverse variation:

(L.B. 2009, 2010)

A relation between two quantities such that an increase in one quantity causes a decrease in the other quantity and decrease in one quantity causes an increase in the other quantity in the same ratio, is called inverse variation.

If two quantities x and y vary inversely,

Then $y \propto \left(\frac{1}{x}\right)$

$y = k(\frac{1}{x})$

$xy = k.$

Examples:

(i) Number of workers and number of days

(ii) Food and men

(iii) Volume and pressure

(iv) Current and resistance

(vi) Fourth Proportional:

If four quantities a, b, c and d are linked as a : b :: c : d then d is called the fourth proportional.

From above

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

$d = \frac{bc}{a}$

(vii) Third Proportional

If three quantities a, b, and c are related as a : b :: b : c then c is called the third proportional.

From above

$ac = b^2$

$c = \frac{b^2}{a}$

(viii) Mean Proportional:

If three quantities a, b, and c are linked as a : b :: b : c then b is called mean proportional.

From above

$ac = b^2$

$b^2 = \pm\sqrt{ac}$

(ix) Continued Proportion:

If three quantities a, b, and c are linked as a : b :: b : c then we say that these quantities are in continued proportion.

From above

$\frac{a}{b} = \frac{b}{c}$

$b^2 = ac$

i.e. in a continued proportion

Q.2 If a : b = c : d then show that b : a = d : c (invertendo theorem)

Ans: let a : b = c : d

$\frac{a}{b} = \frac{c}{d}$

$bc = ad$ Dividing by ac

$\frac{bc}{ac} = \frac{ad}{ac}$

$\frac{b}{a} = \frac{d}{c}$

b : a = d : c

Q.3 If a : b = c : d then show that (Alternendo theorem)

a :c = b: d

Ans: Let a : b = c : d

$\frac{a}{b} = \frac{c}{d}$

$ad = bc$

$\frac{ad}{cd} = \frac{bc}{cd}$ Dividing by cd

$\frac{a}{c} = \frac{b}{d}$

a : c = b : d

Q.4 If a : b = c : d then show that a + b : b = c + d : d (Componendo theorem)

Ans: Let a : b = c : d

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$ adding '1' on both sides

$$\frac{a+b}{b} = \frac{c+d}{d}$$

$(a + b) : b = (c + d) : d$

Q. 5 If a : b = c : d then show that

$a : (a + b) = c : (c + d)$

(Componendo theorem)

Ans: Let a : b = c : d

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{b}{a} = \frac{d}{c}$$ (by invertendo theorem)

$$\frac{b}{a} + 1 = \frac{d}{c} + 1$$ adding '1' both sides

$$\frac{b}{a} + 1 = \frac{d}{c} + 1$$

$$\frac{b+a}{a} = \frac{d+c}{c}$$

$$\frac{a}{b+a} = \frac{c}{d+c}$$ (by invertendo theorem)

$a : (a + b) = c : (c + d)$

Q. 6. If a : b = c : d then show that a – b : b = c – d : d (Dividendo theorem)

Ans: Let a : b = c : d

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$ subtracting '1' both sides

$$\frac{a-b}{b} = \frac{c-d}{d}$$

$(a - b) : b = (c - d) : d$

(ii) a : a – b = c : c – d

Let a : b = c : d then

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{b}{a} = \frac{d}{c}$$

$$\frac{b}{a} - 1 = \frac{d}{c} - 1$$

$$\frac{b-a}{a} = \frac{d-c}{c}$$

$$\frac{a}{b-a} = \frac{c}{d-c}$$

$a : b - a = c : d - c$

$a : a - b = c : c - d$

Q.7 If a : b = c : d then show that (a + b) : (a – b) = (c + d) : (c – d)

Ans: Let a : b = c : d then

$$\frac{(a+b)}{b} = \frac{(c+d)}{d}$$ (i) (By Componendo property)

And $$\frac{a-b}{b} = \frac{c-d}{d}$$ (ii) (By Dividendo property)

dividing (i) by (ii) we get

$$\frac{\frac{a+b}{b}}{\frac{a-b}{b}} = \frac{\frac{c+d}{d}}{\frac{c-d}{d}}$$

$$\frac{a+b}{a-b} = \frac{c+d}{c-d}$$

$a + b : a - b = c + d : c - d$

(Componendo Dividendo Property)

Q.8 What is K–Method?(Lahore Board 2008, 2009)

Ans: Let a:b :: c : d be a Proportion, then

$\frac{a}{b} = \frac{c}{d} = k$ (say)

$\frac{a}{b} = k$ and $\frac{c}{d} = k$

$a = bk, c = dk$

These equations are used to evaluate certain expression more easily. This method is called K–method.

Exercise 3.1

Q.1 For what value of x, the ratio $3 + x : 2 + 4x$ is equal to the ratio 2:3

Sol: Given that

$3+x:2+4x=2:3$

$\frac{3+x}{2+4x}=\frac{2}{3}$

or $9+3x=4+8x$

$4+8x=9+3x$

$8x-3x=9-4$

$5x=5$

$x=1$

Q.2 For what value of m, the ratio 6m+4:8m+2 is equal to the ratio 4:5

Sol: Given that

$6m + 4: 8m + 2 = 4: 5$

or $\frac{6m+4}{8m+2}=\frac{4}{5}$

$\frac{3m+2}{4m+1}=\frac{4}{5}$

By cross multiplication

$15m+10 = 16m+4$

$16m+4 = 15m+10$

$16m - 15m = 10 - 4$

$m = 6$

Q.3 If x : y = 3 : 2, then find 2x : 3y : 4x + 3y

Sol: Given

$x : y = 3:2$

$\frac{x}{y}=\frac{3}{2}$ …… (1)

Now

$2x+3y:4x+3y = \frac{2x+3y}{4x+3y}$

$=\frac{2\left(\frac{x}{y}\right)+3}{4\left(\frac{x}{y}\right)+3}$

(Dividing by y)

$=\frac{2\left(\frac{3}{2}\right)+3}{4\left(\frac{3}{2}\right)+3}$ *for* (1)

$=\frac{3+3}{6+3}$

$=\frac{6}{9}$

$=\frac{2}{3}$

So $2x+3y : 4x+3y = 2 : 3$

Q.4 If a : b = 3 : 4, find 5a+4b : 6a + 9b

Sol: Given $a : b = 3 : 4$

$\Rightarrow \quad \frac{a}{b}=\frac{3}{4} \qquad \text{..... (1)}$

Now

$5a+4b : 6a+9b \quad = \frac{5a+4b}{6a+9b}$

(Dividing by b)

$$=\frac{5\left(\frac{a}{b}\right)+4}{6\left(\frac{a}{b}\right)+9}$$

$$=\frac{5\left(\frac{3}{4}\right)+4}{6\left(\frac{3}{4}\right)+9}$$

$$=\frac{15+16}{18+36}$$

$$=\frac{31}{54}$$

so $5a + 4b : 6a + 9b = 31:54$

Q.5 The Ratio between two numbers is 3 : 4 If 12 is added in both numbers then the new ratio becomes 7 : 8. Find the numbers.

Sol: Since the ratio between number is 3 : 4 Let. The numbers be 3x,4x according to given condition.

$$\frac{3x+12}{4x+12}=\frac{7}{8}$$

By cross multiplication

$28x + 84 = 24x + 96$

$28x - 24x = 96 - 84$

$4x = 12$

$x = 3$

so req. numbers are

$3x = 3(3) = 9$

and $4x = 4(3) = 12$

Q.6 The ratio between two numbers is 3:2. If 8 is subtracted from both numbers then the new ratio becomes 5:6. Find the numbers.

Sol: Since the ratio between the numbers is 3:2

So let the numbers be 3x,2x

According to given condition

$$\frac{3x-8}{2x-8}=\frac{5}{6}$$

By cross multiplication

$18x - 48 = 10x - 40$

$18x - 10x = - 40 + 48$

$8x = 8$

$x = 1$

So the numbers are

$3x = 3(1) = 3$

$2x = 2(1) = 2$

Q.7 Find the value of x for the following.

(i) **2x : 4–x : : 2 : 3**

Sol: Given $2x : 4 - x : : 2 : 3$

As product of extremes = product of means

So $2x\,(3) = (4 - x)(2)$

$6x = 8 - 2x$

$6x + 2x = 8$

$8x = 8$

$x = 1$

(ii) **2x – 1 : 3 : : 5x : 10**

Sol: Given $2x - 1 : 3 : : 5x:10$

$(2x - 1)\,10 = 3(5x)$

$20x - 15x = 10$

$5x = 10$

$x = 2$

(iii) $\frac{x+1}{2}:\frac{3}{2}::\frac{x}{3}:\frac{4}{5}$

Sol: Given eqs are

$$\frac{x+1}{2}:\frac{3}{2}::\frac{x}{3}:\frac{4}{5}$$

$\therefore$ Product of extremes = product of means

$$\text{so } \frac{4}{5}\left(\frac{x+1}{2}\right)=\frac{3}{2}\left(\frac{x}{3}\right)$$

$$\frac{4x+4}{10}=\frac{x}{2}$$

$$\Rightarrow \frac{\cancel{2}(2x+2)}{\cancel{10}_{5}}=\frac{x}{2}$$

$$\frac{2x+2}{5}=\frac{x}{2}$$

By Cross multiplication

$$5x = 4x + 4$$

$$5x - 4x = 4$$

$$x = 4$$

Q.8 Find the unknown term in the following proportion.

(i) $a^2-b^2:a-b::a+b:?$

Sol: Given $a^2-b^2:a-b::a+b:?$

$$\Rightarrow \frac{a^2-b^2}{a-b} = \frac{a+b}{?}$$

$$\frac{(a-b)(a+b)}{(a-b)} = \frac{a+b}{?}$$

$$a+b = \frac{a+b}{?}$$

$$\Rightarrow ? = \frac{a+b}{a+b}$$

so unknown term is $=1$

(ii) $?:a^2+ab+b^2::a^2-ab+b^2:a^3-b^3$

Sol: Given

$$?:a^2+ab+b^2::a^2-ab+b^2:a^3-b^3$$

$$\Rightarrow \frac{?}{a^2+ab+b^2}=\frac{a^2-ab+b^2}{a^3-b^3}$$

$$? = \frac{(a^2-ab+b^2)(a^2+ab+b^2)}{(a-b)(a^2+ab+b^2)}$$

$$? = \frac{a^2-ab+b^2}{a-b}$$

So the unknown term is $\dfrac{a^2-ab+b^2}{a-b}$

(iii) $6x^3y^3z^3:2xyz^2::3x^2yz:?$

Sol: Given $6x^3y^3z^3:2xyz^2::3x^2yz:?$

$$\Rightarrow \frac{6x^3y^3z^3}{2xyz^2}=\frac{3x^2yz}{?}$$

$$\Rightarrow ?(6x^3y^3z^3) = (3x^2yz)(2xyz^2)$$

$$\Rightarrow ? = \frac{(6x^3y^2z^3)}{6x^3y^3z^3} = \frac{1}{y}$$

So the unknown term $= \dfrac{1}{y}$

Q.9 If we add the same number to each of 5,7,17 and 21 then we get proportion in them. Find that numbers.

Sol: Let the req. number be x

According to given condition

$$5 + x : 7 + x : : 17 + x : 21 + x$$

$$\Rightarrow (5 + x)(21 + x) = (7 + x)(17 + x)$$

$$105+ 5x + 21x + x^2 = 119 + 7x +17x + x^2$$

$$x^2 + 26x + 105 = x^2 + 24x + 119$$

$$x^2 + 26x + 105 - x^2 - 24x - 119 = 0$$

$$2x - 14 = 0$$

$$2x = 14$$

$$x = 7$$

So req. number is 7

Q.10 Find the value of x if $17 - x : 31 - x : : 25 - x : 47 - x$

Sol: Given $17 - x : 31 - x :: 25 - x : 47 - x$

$$\Rightarrow \frac{17-x}{31-x}=\frac{25-x}{47-x}$$

$$(17 - x)(47 - x) = (25 - x)(31 - x)$$

$$799 - 17x - 47x + x^2 = 775 - 25x - 31x + x^2$$

$$x^2 - 64x + 799 = x^2 - 56x + 775$$

$$x^2 - 64x + 799 - x^2 + 56x - 775 = 0$$

$$-8x + 24 = 0$$

$$8x = 24$$
$$x = 3$$

So req. number is 3

Q.11 Find the value of x if $\frac{3}{x-1}:\frac{2}{x-1}::\frac{x+1}{3}:\frac{x-2}{4}$.

Sol:

Given

$$\frac{3}{x-1}:\frac{2}{x-1}::\frac{x+1}{3}:\frac{x-2}{4}$$

$$\Rightarrow \left(\frac{3}{x-1}\right)\left(\frac{x-2}{4}\right)=\left(\frac{2}{x-1}\right)\left(\frac{x+1}{3}\right)$$

$$\frac{3x-6}{4x-4}=\frac{2x+2}{3x-3}$$

$$(3x-3)(3x-6)=(2x+2)(4x-4)$$

$$9x^2-18x-9x+18=8x^2-8x+8x-8$$

$$9x^2-27x+18-8x^2+8=0$$

$$x^2-27x+26=0$$

$$x^2-26x-x+26=0$$

$$x(x-26)-1(x-26)=0$$

$$(x-26)(x-1)=0$$

$$\Rightarrow \quad x-26=0 \quad or \quad x-1=0$$

$$x=26 \quad or \quad x=1$$

so req. value of x = 1,26

Exercise 3.2

Q.1 $y \propto x$ and $y = 9$ for $x = 3$ If $x = 4$ then find y.

Sol: As $y \propto x$

$$\Rightarrow \frac{y}{x}=k$$

or $y = kx$ ...... (1)

Put $x = 3,\ y = 9$

$9 = 3k$

$3k = 9$

$k = 3$

Put $k = 3$ and $x = 4$ in 1

$y = 3(4)$

$y = 12$

Q.2 $y = 15$ for $x = 6$ and $y \propto x$, Find the value of y, if $x = 8$

Sol: Given $y \propto x$

$$\Rightarrow \frac{y}{x}=k$$

or $y = kx$ .... (1)

Put $x = 6,\ y = 15$

$15 = 6k$

$6k = 15$

$$k=\frac{5}{2}$$

Now put $k=\frac{5}{2}$, $x = 8$ in 1

$$y=\frac{5}{2}(8)$$

$y = 20$

Q.3 Complete the following table if $y \propto x$.

x	2	4	6			12	
y	4			16	20		28

Sol: As $y \propto x$

$$\Rightarrow \frac{y}{x}=k$$

Put $x = 2,\ y = 4$

So $\frac{4}{2}=k$

$\Rightarrow k = 2$

Here $\frac{y}{x}=2$

or $y = 2x$

Now for $x = 4,\ y = 2(4) = 8$

For $x = 6,\ y = 2(6) = 12$

for $y = 16$, $2x = 16$ or $x = 8$
for $y = 20$, $2x = 20$ or $x = 10$
for $x = 12$, $y = 2(12) = 24$
for $y = 28$, $2x = 28$ or $x = 14$
Here complete table is

x	2	4	6	8	10	12	14
y	4	8	12	16	20	24	28

Q.4 $v \propto t$ and v = 45 m/sec for t = 3s Find v, if t = 2s

Sol: As $v \propto t$
or $v = kt$ ....(1)
Put $v = 45, t = 3$
$45 = k(3)$
$\frac{45}{3} = k$
$\Rightarrow k = 15$
Now Put $k = 15, t = 2$ in 1
$V = 15(2)$
$V = 30$
So $v = 30m / see$

Q.5 Find the value of k if $a \propto b^3$ and a = 250 for b = 5

Sol: As $a \propto b^3$
$\Rightarrow \frac{a}{b^3} = k$ ....(1)
Put $a = 250, b = 5$ in (i)
$b = 5$ in (1)
$\frac{250}{(5)^3} = k$
$\frac{250}{125} = k$
$2 = k$
or $k = 2$

Q.6 If $p \propto q^3$ and p = 128 For q = 4 Find p if q = 5

Sol: As $p \propto q^3$
$\Rightarrow p = kq^3$
$\Rightarrow \frac{p}{q^3} = k$ .... (1)
Put $p = 128, q = 4$ in ....(i)
$\frac{128}{(4)^3} = k$
$\frac{128}{64} = k$
$k = 2$
Put $k = 2$ and $q = 5$ in (1)
$\frac{p}{(5)^3} = 2$
$\frac{p}{125} = 2$
$p = 250$

Q.7 $s \propto t^2$ and s = 80 for t = 4 Find s if t = 3

Sol: As $s \propto t^2$
As $s = kt^2$ .... (1)
Put $s = 80, t = 4$
$80 = k(4)^2$
$80 = 16k$
$\Rightarrow k = 5$
Now put $k = 5$ and $t = 3$ in (1)
$S = 5(3)^2$
$= 5 \times 9$
$S = 45$

Q.8 $a \propto b^2$ and a = 3.6 For b = 3 Find a if b = 2.

Sol: As $a \propto b^2$
$\Rightarrow a = kb^2$
$\Rightarrow \frac{a}{b^2} = k$

As $a = kb^2$ .... (1)

Put $a = 3.6$, and $b = 3$

$3.6 = k(3)^2$

$3.6 = 9k$

$\Rightarrow \quad k = \frac{3.6}{9}$

$k = 0.4$

Put $k = 0.4$ and $b = 2$ in 1

$a = (0.4)(2)^2$

$= (0.4)(4)$

$a = 1.6$

Q.9 $v \propto b^3$ and $v = 32$ for $r = 2$ Find v if $r = 4$

Sol: As $v \propto r^3$

$\Rightarrow \quad v = kr^3$

$\Rightarrow \quad \frac{v}{r^3} = k$

As $v = kr^3$ .... (1)

Put $v = 32$, $r = 2$

$32 = k(2)^3$

$32 = 8k$

$\Rightarrow \quad k = 4$

Now Put $k = 4$, $r = 4$ in 1

$v = 4(4)^3$

$= 4(64)$

$v = 256$

Q.10 $y = 64$ for $x = 16$ Find y for the following conditions if $x = 8$

(i) $y \propto x$

Sol: As $y \propto x$

$\Rightarrow \quad \frac{y}{x} = k$

As $y = kx$ .... (1)

Put $x = 16$, $y = 64$

$64 = 16k$

$\Rightarrow \quad k = 4$

Put $k = 4$, $x = 8$ in (1)

$y = 4(8)$

$y = 32$

(ii) $y \propto \sqrt{x}$

Sol: As $y \propto \sqrt{x}$

$y = k\sqrt{x}$ ...... (1)

Put $x = 16$, $y = 64$

$64 = k\sqrt{16}$

$64 = k(4)$

$k = 16$

Put $k = 16$, $x = 8$ in (1)

$y = 16\sqrt{8}$

$= 16(2\sqrt{2})$

$y = 32\sqrt{2}$

(iii) $y \propto x^2$

Sol: As $y \propto x^2$

$y = kx^2$ ...... (1)

Put $x = 16$, $y = 64$

$64 = k(16)^2$

$64 = 256k$

or $k = \frac{64}{256}$

$K = \frac{1}{4}$

Put $k = 16$, $x = 8$ in (1)

$y = \frac{1}{4}(8)^2$

$= \frac{1}{4}(64)$

$y = 16$

(iv) $\mathbf{y \propto \frac{1}{\sqrt{x}}}$

Sol: As $y \propto \frac{1}{\sqrt{x}}$

$\Rightarrow \quad \frac{y}{\frac{1}{\sqrt{x}}} = k$

or $\quad y = \frac{k}{\sqrt{x}}$ ...... (1)

Put x = 16, y = 64

$64 = \frac{k}{\sqrt{16}}$

$64 = \frac{k}{4}$

$\Rightarrow \quad k = 256$

Put k = 256, x = 8 in (1)

$y = \frac{256}{\sqrt{8}}$

$= \frac{256}{2\sqrt{2}}$

$y = \frac{128}{\sqrt{2}}$

(v) $\mathbf{y \propto x^3}$

Sol: As $y \propto x^3$

$\Rightarrow \quad \frac{y}{x^3} = k$

or $\quad y = kx^3$ ...... (1)

Put x = 16, y = 64

$64 = k(16)^3$

$64 = (16)^2(16)k$

$k = \frac{64}{(16)^2(16)}$

$= \frac{4}{256}$

$k = \frac{1}{64}$

Put $k = \frac{1}{64}$, x = 8 in (1)

$y = \frac{1}{64}(8)^3$

$= \frac{1}{64}(64)(8)$

$y = 8$

Q.11 y ∝ x and y = 16 for x = 4 Find y if x = 6

Sol: As $y \propto x$

$\Rightarrow \quad \frac{y}{x} = k$

or y = kx ...... (1)

Put x = 4, y = 16

16 = k(4)

$\Rightarrow \quad k = 4$

Put k = 4, x = 6 in (1)

y = 4(6)

y = 24

Q.12 Complete the following table if v ∝ t..

v	3	6			15
t	2		6	8	

Sol: As $v \propto t$

$\Rightarrow \quad \frac{v}{t} = k$

or v = kt ...... (1)

Put v = 3, t = 2

3 = 2k

$\Rightarrow \quad k = \frac{3}{2}$

Put in 1

$v = \frac{3}{2}t$

for $v = 6, \quad 6 = \frac{3}{2}t \quad$ *or* $\quad t = 4$

for $t = 6, \quad v = \frac{3}{2}(6) \quad = 9$

for $t = 8, \quad v = \frac{3}{2}(8) \quad = 12$

for $v=15,\ 15=\frac{3}{2}t$ *or* $t=10$

So Complete table is

v	3	6	9	12	15
t	2	4	6	8	10

Exercise 3.3

Q.1 $y\propto\frac{1}{x}$ *and* $y=12$ for x = 5 Find y if $x=2\frac{1}{2}$

Sol: As $y\propto\frac{1}{x}$

$\Rightarrow \quad y=\frac{k}{x}$ …….. (1)

Put x = 5, y = 12

$12=\frac{k}{5}$

$\Rightarrow \quad k=60$

Put k = 60, $x=2\frac{1}{2}=\frac{5}{2}$ in (1)

$y=\frac{60}{5/2}$

$=60\times\frac{5}{2}$

y = 24

Q.2 x and y Varies inversely and k = 20 then find y if x = 5

Sol: As $y\propto\frac{1}{x}$

$\Rightarrow \quad y=\frac{k}{x}$ …….. (1)

Put k = 20, x = 5 in (1)

$y=\frac{20}{5}$

y = 4

Q.3 $y\propto\frac{1}{x}$ *and* $y=12$ **for x = 4 Find y if x = 6**

Sol: As $y\propto\frac{1}{x}$

$\Rightarrow \quad y=\frac{k}{x}$ …….. (1)

Put x = 4, y = 12

$12=\frac{k}{4}$

$\Rightarrow \quad k=48$

Put k = 48, x = 6 in (1)

$y=\frac{48}{6}$

y = 8

Q.4 $y\propto\frac{1}{x}$ *and* $y=12$ **for x = 6 Find y if**

(i) x = 3

Sol: As $y\propto\frac{1}{x}$

$\Rightarrow \quad y=\frac{k}{x}$ …….. (1)

Put x = 6, y = 12

$12=\frac{k}{6}$

$k=72$

Put k = 72, x = 3 in (1)

$y=\frac{72}{3}$

y = 24

(ii) **x = 4**

Sol: As $y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ …….. (1)

Put x = 6, y = 12

$12 = \frac{k}{6}$

$\Rightarrow \quad k = 72$

Put k = 72, x = 4 in (1)

$y = \frac{72}{4}$

y = 18

Q.5 $y \propto \frac{1}{x}$ *and* $y = 4$ for $x = \frac{1}{2}$

Find y if $x = \frac{5}{2}$

Sol: As $y \propto \frac{1}{x}$

$\Rightarrow \quad y = \frac{k}{x}$ …….. (1)

Put $x = \frac{1}{2}$, y = 4 in (1)

$4 = \frac{k}{\frac{1}{2}}$

$k = 4 \times \frac{1}{2}$

k = 2

Put k = 2, $x = \frac{5}{2}$ in (1)

$y = \frac{2}{\frac{5}{2}}$

$y = \frac{4}{5}$

Q.6 $n \propto \frac{1}{r^2}$ *and* $n = 4$ for r = 6 Find n if r = 10

Sol: As $n \propto \frac{1}{r^2}$

$\Rightarrow \quad n = \frac{k}{r^2}$ …….. (1)

Put n = 4, r = 6

$4 = \frac{k}{(6)^2}$

$4 = \frac{k}{36}$

$\Rightarrow \quad k = 144$

Put k = 144, r = 10 in 1

$n = \frac{144}{(10)^2}$

$n = \frac{144}{100}$

n = 1.44

Q.7 $\mathbf{s \propto \frac{1}{t^2}}$ **and s = 4 for t = 2 Find s if t = 3**

Sol: As $s \propto \frac{1}{t^2}$

$\Rightarrow \quad s = \frac{k}{t^2}$ …….. (1)

Put t = 2, s = 4

$4 = \frac{k}{(2)^2}$

$4 = \frac{k}{4}$

$\Rightarrow \quad k = 16$

Put k = 16, t = 3 in (1)

$s = \frac{16}{(3)^2}$

$s = \frac{16}{9}$

Q.8 $y \propto \frac{1}{x^2}$ *and* $y = 8$ **for x = 2**

Find S if t = 3

Sol: As $y \propto \frac{1}{x^2}$

$\Rightarrow \quad y = \frac{k}{x^2} \quad \text{........ (1)}$

Put x = 2, y = 8

$8 = \frac{k}{(2)^2}$

$8 = \frac{k}{4}$

$\Rightarrow \quad k = 32$

Put k = 32, x = 4 in (1)

$y = \frac{32}{(4)^2}$

$y = \frac{32}{16}$

y = 2

Q.9 $p \propto \frac{1}{q^2}$ *and* $p = 2$ **for q = 5 Find q if p = 2**

Sol: As $p \propto \frac{1}{q^3}$

$\Rightarrow \quad p = \frac{k}{q^3} \quad \text{........ (1)}$

Put p = 2, q = 5

$2 = \frac{k}{(5)^3}$

$2 = \frac{k}{125}$

$\Rightarrow \quad k = 250$

Put k = 250, p = 2 in (1)

$2 = \frac{250}{q^3}$

$q^3 = \frac{250}{2}$

$q^3 = 125$

$q^3 = (5)^3$ (Taking cube root)

$\Rightarrow$ q = 5

Q.10 $a \propto \frac{1}{b^3}$ *and* $a = 8$ for b = 3

Find a if b = 2

Sol: As $a \propto \frac{1}{b^3}$

$\Rightarrow \quad a = \frac{k}{b^3} \quad \text{........ (1)}$

Put a = 8, b = 3

$8 = \frac{k}{(3)^3}$

$8 = \frac{k}{27}$

$\Rightarrow \quad k = 216$

Put k = 216, b = 2 in 1

$a = \frac{216}{(2)^3}$

$= \frac{216}{8}$

a = 27

Exercise 3.4

Q.1 Find the fourth proportional in the following.

(i) 7,21,3

Sol: Let x be the fourth proportional then 7 : 21 :: 3 : x

$\Rightarrow \quad 7x = 21\times 3$

$7x = 63$

$x = 9$

So the fourth proportional is 9

(ii) $3a^2b^2, 5ab^3, 9ab$

Sol: Let x be the fotrth proportional then $3a^2b^2 : 5ab^3 :: 9ab : x$

$\Rightarrow \quad 3a^2b^2x = 5ab^3\times 9ab$

$$x = \frac{45a^2b^3}{3a^2b^2}$$

$x = 15b$

So the fourth proportional is 15b

(iii) $a^2+ab+b^2, a^3-b^3, a+b$

Sol: Let x be the fourth proportional

$\Rightarrow \quad a^2+ab+b^2 : a^3-b^3 :: a+b : x$

$\Rightarrow \quad (a^2+ab+b^2)x = (a^3-b^3)(a+b)$

$$x = \frac{(a-b)(a^2+ab+b^2)(a+b)}{(a^2+ab+b^2)}$$

$x = (a-b)(a+b)$

$x = a^2-b^2$

(iv) $l^2-m^2, (l+m)^2, (l-m)(l^2-lm+m^2)$

Sol: Let x be the fourth proportional, then

So

$l^2-m^2 : (l+m)^2 :: (l-m)(l^2-lm+m^2) : x$

$\Rightarrow (l^2-m^2)x = (l+m)^2(l-m)(l^2-lm+m^2)$

$$x = \frac{(l+m)^2(l-m)(l^2-lm+m^2)}{(l^2-m^2)}$$

$$= \frac{(l+m)^2(l-m)(l^2-lm+m^2)}{(l-m)(l+m)}$$

$= (l+m)(l^2-lm+m^2)$

$x = l^3+m^3$

So fourth proportional is l^3+m^3

(v) $x^2-5x+6, x-3, x^2-4$

Sol: Let x be the fourth proportional, then

$x^2-5x+6, x-3, x^2-4 : a$

$\Rightarrow \quad (x^2-5x+6)a = (x-3)(x^2-4)$

$(x-2)(x-3)a = (x-3)(x^2-4)$

$$a = \frac{(x-3)(x-2)(x+2)}{(x-2)(x-3)}$$

$a = x+2$

So fourth proportional is $x+2$

Q.2 Find the third proportional in the following.

(i) 3,12

Sol: Let x be the third proportional, then

3 : 12 :: 12 : x

$\Rightarrow \quad 3x = (12)(12)$

$3x = 144$

$$x = \frac{144}{3}$$

$x = 48$

So the third proportional is 48

(ii) $75a^4b^5, 15a^7b^9$

Sol: Let x be the third proportional, then

$75a^4b^5 : 15a^7b^9 :: 15a^7b^9 : x$

$\Rightarrow \quad 75a^4b^5x = (15a^7b^9)(15a^7b^9)$

$$x = \frac{225a^{14}b^{18}}{75a^4b^5}$$

$x = 3a^{10}b^{13}$

So the third proportional is $3a^{10}b^{13}$

(iii) $(a-2)^2, a^2-a-2$

Sol: Let x be the third proportional, then

$(a-2)^2 : a^2-a-2 : a^2-a-2 : x$

$\Rightarrow \quad (a-2)^2 x = (a^2-a-2)(a^2-a-2)$

$(a-2)^2 x = (a-2)(a+1)(a-2)(a+1)$

$$x = \frac{(a-2)(a+1)(a-2)(a+1)}{(a-2)(a-2)}$$

$\Rightarrow \quad x = (a+1)^2$

So the third proportional is $(a+1)^2$

(iv) $x+y, x^2-y^2$

Sol: Let x be the third proportional, then

$x+y : x^2-y^2 :: x^2-y^2 : a$

$\Rightarrow \quad (x+y)a = (x^2-y^2)^2$

$\Rightarrow \quad (x+y)a = (x^2-y^2)(x^2-y^2)$

$$a = \frac{(x+y)(x-y)(x+y)(x-y)}{(x+y)}$$

$a = (x+y)(x-y)^2$

So the third proportional is $(x+y)(x-y)^2$

(v) $2a^2-ab-6b^2, 2a+3b$

Sol: Let x be the third proportional then $2a^2-ab-6b^2 : 2a+3b :: 2a+3b : x$

$\Rightarrow \quad (2a^2-ab-6b^2)x = (2a+3b)(2a+3b)$

$$x = \frac{(2a+3b)(2a-3b)}{(2a^2-ab-6b^2)}$$

$$x = \frac{(2a+3b)(2a-3b)}{(2a+3b)(a-2b)}$$

So the third proportional is

$$x = \frac{2a-3b}{a-2b}$$

Q.3 Find the mean proportional in the following:

(i) 16,9

Sol: Let m be the mean proportional then 16 : m :: m : 9

$\Rightarrow \quad (16)(9) = m^2$

$m^2 = 144$

$m = \pm 12$

So the mean proportional is ± 12

(ii) a^5, a^9

Sol: Let m be the mean proportional then $a^5 : m :: m : 9$

$\Rightarrow \quad m^2 = (a^5)(a^9)$

$m^2 = a^{14}$

$\sqrt{m^2} = \sqrt{(\pm a^7)^2}$

$m = \pm a^7$

So the mean proportional is a^7

(iii) $(x-2), (x+3)(x^2+x-6)$

Sol: Let m be the mean proportional then $x-2 : m :: m : (x+3)(x^2+x-6)$

$\Rightarrow \quad m^2 = (x-2)(x+3)(x^2+x-6)$

$= (x^2+x-6)(x^2+x-6)$

$m^2 = (x^2+x-6)^2$

$\sqrt{m^2} = \sqrt{(x^2+x-6)^2}$

$m = \pm(x^2+x-6)$

(iv) $\dfrac{x^7}{y^3}, \dfrac{y^3}{x}$

Sol: Let m be the mean proportional then $\dfrac{x^7}{y^3} : m :: m : \dfrac{y^3}{x}$

$$\Rightarrow \quad m^2 = \frac{x^7}{\cancel{y^3}} \; \frac{\cancel{y^3}}{x}$$

$m^2 = x^6$

$\sqrt{m^2} = \sqrt{(\pm x^3)^2}$

$m = \pm x^3$

So the mean proportional is $+ x^3$

(v) $\frac{a-b}{a+b}, \frac{a^2b^2}{a^2-b^2}$

Sol: Let m be the mean proportional

then $\frac{a-b}{a+b} : m :: m : \frac{a^2b^2}{a^2-b^2}$

$$\Rightarrow \quad m^2 = \left(\frac{a-b}{a+b}\right)\left(\frac{a^2b^2}{a^2-b^2}\right)$$

$$= \frac{(a-b)a^2b^2}{(a+b)(a+b)(a-b)}$$

$$m^2 = \frac{a^2b^2}{(a+b)^2}$$

$$or \quad \sqrt{m^2} = \sqrt{\left(\frac{\pm ab}{a+b}\right)^2}$$

$$m = \pm\left(\frac{ab}{a+b}\right)$$

Q.4 Find the unknown in the following continued proportions

(i) 4, x, 16

Sol: Since there is a continued proportion

so $4 : x :: x : 16$

$\Rightarrow \quad x^2 = 64$

$x = \pm 8$

(ii) 9, m - 1, 16

Sol: Since there is a continued proportion

so $9 : m-1 :: m-1 : 16$

$\Rightarrow \quad (m-1)^2 = 9(16)$

$(m-1)^2 = 144$

$m-1 = \pm 12$

$m = 1 \pm 12$

$m = 1+12 \quad or \quad m = 1-12$

$m = 13 \quad or \quad m = -11$

(iii) 3, x + 2, 27

Sol: Since there is a continued proportions

so $3 : x+2 :: x+2 : 27$

$\Rightarrow \quad (x+2)^2 = 3(27)$

$(x+2)^2 = 81$

$x+2 = \pm 9$

$x = -2 \pm 9$

$= -2+9 \quad or \quad -2-9$

$x = 7 \quad or \quad -11$

Exercise 3.5

Q.1 If a : b = c : d then prove that

(i) 5a + 4b : 5a – 4b = 5c + 4d : 5c – 4d

Sol:

As $a : b = c : d$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$

Multiplying both sides by $\frac{5}{4}$

$$\frac{5a}{4b} = \frac{5c}{4d}$$

(by componendo – dividendo theorem)

$$\frac{5a+4b}{5a-4b} = \frac{5c+4d}{5c-4d}$$

or 5a + 4b : 5a – 4b = 5c + 4d : 5c – 4d

(ii) 3a + 5b : 3c + 5d = 3a – 5b : 3c – 5d

Sol:

As $a : b = c : d$

$\Rightarrow \quad \frac{a}{b}=\frac{c}{d}$

Multiplying both sides by $\frac{3}{5}$

$\frac{3a}{5b}=\frac{3c}{5d}$

(by componendo – dividend theorem)

$\frac{3a+5b}{3a-5b}=\frac{3c+5d}{3c-5d}$

or $\frac{3a+5b}{3c+5d}=\frac{3a-5b}{3c-5d}$

or $3a+5b : 3c+5d = 3a-5b : 3c-5d$

(iii) $\frac{a^2+b^2}{a^2-b^2}=\frac{c^2+d^2}{c^2-d^2}$

Sol:

As $a:b=c:d$

$\Rightarrow \quad \frac{a}{b}=\frac{c}{d}$

Squaring both sides

$\frac{a^2}{b^2}=\frac{c^2}{d^2}$

(by componendo – dividendo theorem)

$\frac{a^2+b^2}{a^2-b^2}=\frac{c^2+d^2}{c^2-d^2}$

(iv) sa + tb : sa – tb = sc + td : sc – td

Sol:

As $a:b=c:d$

$\Rightarrow \quad \frac{a}{b}=\frac{c}{d}$

Multiplying both sides by $\frac{s}{t}$

$\frac{sa}{tb}=\frac{sc}{td}$

(by componendo – dividend theorem)

$\frac{sa+tb}{sa-tb}=\frac{sc+td}{sc-td}$

sa + tb : sa – tb = sc + td : sc – td

(v) $\frac{2a^2+3b^2}{2c^2+3d^2}=\frac{2a^2-3b^2}{2c^2-3d^2}$

Sol:

As $a:b=c:d$

$\Rightarrow \quad \frac{a}{b}=\frac{c}{d}$ squaring

$\Rightarrow \quad \frac{a^2}{b}=\frac{c^2}{d^2}$

Multiplying both sides by

$\frac{2a^2}{3b^2}=\frac{2c^2}{3d^2}$

(by componendo – dividendo theorem)

$\frac{2a^2+3b^2}{2a^2-3b^2}=\frac{2c^2+3d^2}{2c^2-3d^2}$

$\frac{2a^2+3b^2}{2a^2+3d^2}=\frac{2a^2-3b^2}{2c^2-3d^2}$ (By alternendo property

Q.2 Prove that x : y = u : v if

(i) $\frac{3x+7y}{3x-7y}=\frac{3u+7v}{3u-7v}$

Sol: Given $\frac{3x+7y}{3x-7y}=\frac{3u+7v}{3u-7v}$

By componendo dividendo theorem

$\frac{(3x+7y)+(3x-7y)}{(3x+7y)-(3x-7y)}=\frac{(3u+7v)+(3u+7v)}{(3u-7v)-(3u-7v)}$

$\frac{3x+7y+3x-7y}{3x+7y-3x+7y}=\frac{3u+7v+3u-7v}{3u+7v-3u+7v}$

$\frac{6x}{14y}=\frac{6u}{14v}$

$\frac{3x}{7y}=\frac{3u}{7v}$

Multiplying both sides by $\frac{7}{3}$

$$\frac{x}{y}=\frac{u}{v}$$

$$\Rightarrow \quad x:y=u:v$$

(ii)

$$mx+ny:mx-ny=mu+nv:mu-nv$$

Sol: Given

$$mx+ny:mx-ny=mu+nv:mu-nv$$

$$\frac{mx+ny}{mx-ny}=\frac{mu+nv}{mu-nv}$$

By Componendo – dividendo theorem

$$\frac{(mx+ny)+(mx-ny)}{(mx+ny)-(mx-ny)}=\frac{(mu+nv)+(mu-nv)}{(mu+nv)-(mu-nv)}$$

$$\frac{mx+ny+mx-ny}{mx+ny-mx+ny}=\frac{mu+nv+mu-nv}{mu-nv-mu+nv}$$

$$\frac{2mx}{2ny}=\frac{2mu}{2nv}$$

$$\frac{2mx}{2ny}\times\frac{2n}{2m}=\frac{2mu}{2nv}\times\frac{2n}{2m} \quad \text{Multipliny both sides by } \frac{2n}{2m}$$

$$\frac{x}{y}=\frac{u}{v}$$

$$\Rightarrow \quad x:y=u:v$$

(iii) $$\frac{x+y+u+v}{x+y-u-v}=\frac{x-y+u-v}{x-y-u+v}$$

Sol: Given $$\frac{x+y+u+v}{x+y-u-v}=\frac{x-y+u-v}{x-y-u+v}$$

By componendo – dividendo theorem

$$\frac{x+y+u+v+x+y-u-v}{x+y+u+v-x-y+u+v}=\frac{x-y+u-v+x-y-u+v}{x-y+u-v-x+y+u-v}$$

$$\frac{2x+2y}{2u+2v}=\frac{2x-2y}{2u-2v}$$

$$\Rightarrow \frac{2(x+y)}{2(u+v)}=\frac{2(x-y)}{2(u-v)}$$

$$\frac{x+y}{u+v}=\frac{x-y}{u-v}$$

$$\frac{x+y}{x-y}=\frac{u+v}{u-v}$$

Again by componendo – dividendo theorem

$$\frac{x+y+x-y}{x+y-x+y}=\frac{u+v+u-v}{u+v-u+v}$$

$$\frac{2x}{2y}=\frac{2u}{2v}$$

$$\Rightarrow \quad x:y=u:v$$

(iv) $$\frac{4x+5y+4u+5v}{4x+5y-4u-5v}=\frac{4x-5y+4u-5v}{4x-5y-4x+5v}$$

Sol: **Given**

$$\frac{4x+5y+4u+5v}{4x+5y-4u-5v}=\frac{4x-5y+4u-5v}{4x-5y-4u+5v}$$

By componendo – dividendo theorem

$$=\frac{4x+5y+4u+5v+4x+5y-4u-5v}{4x+5y+4u+5v+4x-5y+4u+5v}$$

$$=\frac{4x-5y+4u-5v+4x-5y-4u+5v}{4x-5y+4u-5v-4x+5y+4u-5v}$$

$$\frac{8x+10y}{8u+10v}=\frac{8x-10y}{8u-10v}$$

$$\frac{4x+5y}{4u+5v}=\frac{4x-5y}{4u-5v}$$

$$\frac{4x+5y}{4x-5y}=\frac{4u+5v}{4u-5v}$$

Again by componendo – dividendo theorem

$$\frac{4x+5y+4x-5y}{4x+5y-4x+5y}=\frac{4u+5v+4u-5v}{4u+5v-4u+5v}$$

$$\frac{8x}{10y}=\frac{8u}{10v}$$

Multiply by $\frac{10}{8}$

$$\Rightarrow \quad x:y=u:v$$

(v) $$\frac{xu+yv}{xu-yv}=\frac{x^2+y^2}{x^2-y^2}$$

Sol: **Given:**

$$\frac{xu+yv}{xu-yv}=\frac{x^2+y^2}{x^2-y^2}$$

By componendo – dividendo theorem

$$\frac{xu+yv+xu-yv}{xu+yv-xu+yv}=\frac{x^2+y^2+x^2-y^2}{x^2+y^2-x^2+y^2}$$

$$\frac{2xu}{2yv}=\frac{2x^2}{2y^2}$$

$$\frac{xu}{yv}=\frac{x^2}{y^2}$$

$\frac{u}{v}=\frac{x}{y}$ Multiplying by $\frac{y}{x}$

or $\frac{x}{v}=\frac{u}{v}$

$\Rightarrow \quad x:y=u:v$

Q.3 If $p=\frac{2ab}{a+b}$, then find the value of $\frac{p+a}{p-a}+\frac{p+b}{p-b}$, using componendo-dividendo theorem.

Sol: Given $p=\frac{2ab}{a+b}$

or $\frac{p}{a}=\frac{2b}{a+b}$

Applying componendo–dividendo theorem

$$\frac{p+a}{p-a}=\frac{2b+a+b}{2b-a-b}$$

$$\frac{p+a}{p-a}=\frac{a+3b}{b-a} \qquad \text{.......(1)}$$

Now $\frac{p}{b}=\frac{2a}{a+b}$

Again applying componendo – dividendo theorem

$$\frac{p+b}{p-b}=\frac{2a+a+b}{2a-a-b}$$

$$\frac{p+b}{p-b}=\frac{3a+b}{a-b} \qquad \text{..... (2)}$$

Adding (1) and (2)

$$\frac{p+a}{p-a}+\frac{p+b}{p-b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$$

$$=\frac{a+3b}{b-a}-\frac{3a+b}{b-a}$$

$$=\frac{a+3b-3a-b}{b-a}$$

$$=\frac{2b-2a}{b-a}$$

$$=\frac{2(b-a)}{(b-a)}$$

$$=2$$

Q.4 If $s=\frac{6ab}{a-b}$, then find the value of $\frac{s-3a}{s+3a}+\frac{s+3b}{s-3b}$, using compenendo-dividendo theorem.

Sol: Given $s=\frac{6ab}{a-b}$

$$\frac{s}{3a}=\frac{2b}{a-b}$$

By componendo – dividendo theorem

$$\frac{s+3a}{s-3a}=\frac{2b+a-b}{2b-a+b}=\frac{a+b}{3b-a}$$

$$\frac{s-3a}{s+3a}=\frac{3b-a}{a+b} \qquad \text{.......(1)}$$

Now for given, consider

$$\frac{s}{3b}=\frac{2a}{a-b}$$

Again by componendo–dividendo theorem

$$\frac{s+3b}{s-3b}=\frac{2a+a-b}{2a-a+b}$$

$$\frac{s+3b}{s-3b}=\frac{3a-b}{a+b} \qquad \text{..... (2)}$$

Adding (1) and (2)

$$\frac{s-3a}{s+3a}+\frac{s+3b}{s-3b}=\frac{3b-a}{a+b}+\frac{3a-b}{a+b}$$

$$=\frac{3b-a+3a-b}{a+b}$$

$$=\frac{2b+2a}{a+b}=\frac{2(a+b)}{a+b}$$

$$\frac{s-3a}{s+3a}+\frac{s+3b}{s-3b}=2$$

Q.5 If $x=\frac{8ab}{a+b}$, then find the value of $\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}$, using componendo-dividendo theorem.

Sol: Given $x=\frac{8ab}{a+b}$

$$\frac{x}{4a}=\frac{2b}{a+b}$$

By componendo – dividendo theorem

$$\frac{x+4a}{x-4a}=\frac{2b+a+b}{2b-a-b}$$

$$\frac{x+4a}{x+4a}=\frac{a+3b}{b-a} \quad \text{.......(1)}$$

Now for given

$$\frac{x}{4b}=\frac{2a}{a+b}$$

Again by componendo–dividendo theorem

$$\frac{x+4b}{x-4b}=\frac{2a+a+b}{2a-a-b}$$

$$\frac{x+4b}{x-4b}=\frac{3a+b}{a-b} \quad \text{..... (2)}$$

Adding (1) and (2)

$$\frac{x+4a}{x-4a}+\frac{x+4b}{x-4b}=\frac{a+3b}{b-a}+\frac{3a+b}{a-b}$$

$$=\frac{a+3b}{b-a}-\frac{3a+b}{b-a}$$

$$=\frac{a+3b-3a-b}{(b-a)}$$

$$=\frac{2b-2a}{b-a}$$

$$=\frac{2(b-a)}{(b-a)}$$

$$=2$$

Q.6 Solve the following by using componendo - dividendo property.

(i) $\frac{(x-4)^2+(x-3)^2}{(x-4)^2-(x-3)^2}=\frac{5}{4}$

Sol: Applying componendo – dividendo theorem

$$\frac{(x-4)^2+(x-3)^2+(x-4)^2-(x-3)^2}{(x-4)^2+(x-3)^2-(x-4)^2+(x-3)^2}=\frac{5+4}{5-4}$$

$$\frac{2(x-4)^2}{2(x-3)^2}=9$$

$$\frac{(x-4)^2}{(x-3)^2}=(\pm 3)^2$$

$$\frac{(x-4)}{(x-3)}=\pm 3$$

$$\frac{x-4}{x-3}=3 \quad or \quad \frac{x-4}{x-3}=-3$$

$$3x-9=x-4 \quad or \quad x-4=-3x+9$$

$$3x-x=-4+9 \quad or \quad x+3x=4+9$$

$$2x=5 \quad or \quad 4x=13$$

$$x=\frac{5}{2} \quad or \quad x=\frac{13}{4}$$

so S.S. $=\left\{\frac{5}{2},\frac{13}{4}\right\}$

(ii) $\frac{\sqrt{x^2+8a^2}-\sqrt{x^2+a^2}}{\sqrt{x^2+8a^2}+\sqrt{x^2-a^2}}=\frac{1}{3}$

Sol: Applying componendo – dividendo theorem

$$\frac{\sqrt{x^2+8a^2}-\sqrt{x^2-a^2}+\sqrt{x^2-8a^2}+\sqrt{x^2-a^2}}{\sqrt{x^2+8a^2}-\sqrt{x^2-a^2}-\sqrt{x^2+8a^2}-\sqrt{x^2-a^2}}=\frac{1+3}{1-3}$$

$$\frac{2\sqrt{x^2+8a^2}}{-2\sqrt{x^2-a^2}}=\frac{4}{-2}$$

$$\frac{\sqrt{x^2+8a^2}}{\sqrt{x^2-a^2}}=2$$

Squaring both sides

$$\frac{x^2+8a^2}{x^2-a^2}=4$$

$$4x^2-4a^2=x^2+8a^2$$

$$4x^2-x^2=8a^2+4a^2$$

$$3x^2=12a^2$$

$$x^2=4a^2$$

$$x=\pm 2a$$

So S.S is $\{2a,-2a\}$

(iii) $\dfrac{(x+5)^3-(x-3)^3}{(x+5)^3+(x-3)^3}=\dfrac{13}{14}$

Sol: Applying componendo – dividendo theorem

$$\frac{(x+5)^3-(x-3)^3+(x+5)^3+(x-3)^3}{(x+5)^3-(x-3)^3-(x+5)^3-(x-3)^3}=\frac{13+14}{13-14}$$

$$\frac{2(x+5)^3}{-2(x-3)^3}=\frac{27}{-1}$$

$$\frac{(x+5)^3}{(x-3)^3}=27$$

$$\frac{(x+5)^3}{(x-3)^3}=(3)^3$$

$$\frac{x+5}{x-3}=3$$

$$3x-9=x+5$$

$$3x-x=5+9$$

$$2x=14$$

$$x=7$$

So S.S $=\{7\}$

(iv) $\dfrac{\sqrt{x^2+a^2}+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}=2$

Sol: Applying By componendo – dividendo theorem

$$\frac{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}+\sqrt{x^2+a^2}-\sqrt{x^2-a^2}}{\sqrt{x^2+a^2}+\sqrt{x^2-a^2}-\sqrt{x^2+a^2}+\sqrt{x^2-a^2}}=\frac{2+1}{2-1}$$

$$\frac{2\sqrt{x^2+a^2}}{2\sqrt{x^2-a^2}}=3$$

$$\frac{\sqrt{x^2+a^2}}{\sqrt{x^2-a^2}}=3$$

Squaring both sides

$$\frac{x^2+a^2}{x^2-a^2}=9$$

$$9x^2-9a^2=x^2+a^2$$

$$9x^2-x^2=a^2+9a^2$$

$$8x^2=10a^2$$

$$x^2=\frac{5}{4}a^2$$

$$x=\pm\frac{\sqrt{5}a}{2}$$

So S.S $=\left\{\pm\dfrac{\sqrt{5}a}{2}\right\}$

Exercise 3.6

Q.1 If $\dfrac{a}{b}=\dfrac{c}{d}$ (where $a,b,c,d\neq 0$), **the prove that**

(i) $\dfrac{5a+7b}{5a-7b}=\dfrac{5c+7d}{5c-7d}$

Sol:

Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

$$\text{L.H.S.}=\frac{5a+7b}{5a-7b}=\frac{5(bk)+7b}{5(bk)-7b}=\frac{b(5k+7)}{b(5k-7)}$$

$$=\frac{5k+7}{5k-7} \quad \ldots\ldots (1)$$

$$\text{R.H.S.} = \frac{5c+7d}{5c-7d} = \frac{5dk+7d}{5dk-7d} = \frac{d(5k+7)}{d(5k-7)}$$

$$=\frac{5k+7}{5k-7} \quad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(ii) $$\frac{3a^2-7b^2}{3a^2+7b^2} = \frac{3c^2-7d^2}{3c^2+7d^2}$$

Sol: Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow$ a = bk

c = dk

Then

$$\text{L.H.S.} = \frac{3a^2-7b^2}{3a^2+7b^2} = \frac{3b^2k^2-7b^2}{3b^2k^2+7b^2}$$

$$=\frac{b^2(3k^2-7)}{b^2(3k^2+7)}$$

$$=\frac{3k^2-7}{3k^2+7} \quad \ldots\ldots (1)$$

$$\text{R.H.S.} = \frac{3c^2-7d^2}{3c^2+7d^2} = \frac{3d^2k^2-7d^2}{3d^2k^2+7d^2}$$

$$=\frac{d^2(3k^2-7)}{d^2(3k^2+7)}$$

$$=\frac{3k^2-7}{3k^2+7} \quad \ldots\ldots (2)$$

From (1) and (2)

L.H.S. = R.H.S.

(iii) $$\frac{c^2+d^2}{a^2+b^2} = \frac{cd}{ab}$$

Sol: Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow$ a = bk

c = dk

Now

$$\text{L.H.S.} = \frac{c^2+d^2}{a^2-b^2} = \frac{d^2k^2+d^2}{b^2k^2+b^2}$$

$$=\frac{d^2\,\cancel{(k^2+1)}}{b^2\,\cancel{(k^2+1)}}$$

$$\frac{c^2+d^2}{a^2+b^2} = \frac{d^2}{b^2} \quad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = \frac{cd}{ab} = \frac{dkd}{bkb}$$

$$\frac{d^2k}{b^2k} = \frac{d^2}{b^2} \quad \ldots\ldots (2)$$

from (1) and (2)

$$\frac{c^2+d^2}{a^2+b^2} = \frac{cd}{ab}$$

(iv) $$\frac{a^2+ab+b^2}{c^2+cd+d^2} = \frac{a^2-ab+b^2}{c^2-cd+d^2}$$

Sol: Let $\frac{a}{b} = \frac{c}{d} = k$

$\Rightarrow$ a = bk

c = dk

So

$$\text{L.H.S.} = \frac{a^2+ab+b^2}{c^2+cd+d^2} = \frac{b^2k^2+bk.b+b^2}{d^2k^2+dkd+d^2}$$

$$=\frac{b^2\,\cancel{(k^2+k+1)}}{d^2\,\cancel{(k^2+k+1)}}$$

$$\frac{a^2+ab+b^2}{c^2+cd+b^2} = \frac{b^2}{d^2} \quad \ldots\ldots (1)$$

Now

$$\text{R.H.S} = \frac{a^2-ab+b^2}{c^2-cd+d^2} = \frac{b^2k^2-bk.b+b^2}{d^2k^2-dk.d+d^2}$$

$$=\frac{b^2\,\cancel{(k^2-k+1)}}{d^2\,\cancel{(k^2-k+1)}}$$

$$=\frac{b^2}{d^2} \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(v) $\dfrac{a^2-ab+b^2}{c^2-cd+d^2}=\dfrac{(a-b)^2}{(c-d)^2}$

Sol: Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

So

$$\text{L.H.S.} = \frac{a^2-ab+b^2}{c^2-cd-d^2}=\frac{b^2k^2-bk.b+b^2}{d^2k^2-dk.d+d^2}$$

$$=\frac{b^2\cancel{(k^2-k+1)}}{d^2\cancel{(k^2-k+1)}}$$

$$=\frac{b^2}{d^2} \qquad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = \frac{(a-b)^2}{(c-d)^2}=\frac{(bk-b)}{(dk-d)^2}$$

$$\frac{(a-b)^2}{(c-d)^2}=\frac{b^2\cancel{(k-1)^2}}{d^2\cancel{(k-1)^2}}$$

$$=\frac{b^2}{d^2} \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(vi) $(a-d)^2-(b-c)^2=(a+d)^2-(b+c)^2$

Sol: Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

So

L.H.S.= $(a-d)^2-(b-c)^2=(bk-d)^2-(b-dk)^2$

$$=b^2k^2+d^2-2b\cancel{d}k-b^2-d^2k^2+2\cancel{b}dk$$

$$=b^2k^2-b^2-d^2k^2+d^2$$

$$=b^2(k^2-1)-d^2(k^2-1)$$

$$=(k^2-1)(b^2-d^2) \qquad \ldots\ldots (1)$$

Now

R.H.S. = $(a+d)^2-(b+c)^2=(bk+d)^2-(b+dk)^2$

$$=b^2k^2+d^2+2b\cancel{d}k-b^2-d^2k^2-2\cancel{b}dk$$

$$=b^2k^2-b^2-d^2k^2+d^2$$

$$=b^2(k^2-1)-d^2(k^2-1)$$

$$=(k^2-1)(b^2-d^2) \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(vii) $\dfrac{pa+qc}{pb+qd}=\sqrt{\dfrac{a^2+c^2}{b^2+d^2}}$

Sol: Let $\dfrac{a}{b}=\dfrac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

Now

$$\text{L.H.S.} = \frac{pa+qc}{pb+qd}=\frac{pbk+qdk}{pb+qd}$$

$$=\frac{k(pb+qd)}{(pb+qd)}$$

$$=k \qquad \ldots\ldots (1)$$

$$\text{R.H.S.} = \sqrt{\frac{a^2+c^2}{b^2+d^2}}=\sqrt{\frac{b^2k^2+d^2k^2}{b^2+d^2}}$$

$$=\sqrt{\frac{k^2(b^2+d^2)}{(b^2+d^2)}}$$

$$=\sqrt{k^2}$$

$$=k \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(viii) $\frac{a+b}{c+d}=\sqrt{\frac{a^2-ab+b^2}{c^2-cd+d^2}}$

Sol: Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

Now

L.H.S. $=\frac{a+b}{c+d}=\frac{bk+b}{dk+d}$

$=\frac{b\cancel{(k+1)}}{d\cancel{(k+1)}}$

$=\frac{b}{d}$ ...... (1)

Now

R.H.S. $=\sqrt{\frac{a^2-ab+b^2}{c^2-cd+d^2}}=\sqrt{\frac{b^2k^2-b^2k+b^2}{d^2k^2-d^2k+d^2}}$

$=\sqrt{\frac{b^2(k^2-k+1)}{b^2(k^2-k+1)}}$

$=\sqrt{\frac{b^2}{d^2}}$

$=\frac{b}{d}$ .... (2)

from (1) and (2)

L.H.S. = R.H.S.

(ix) $\left(\frac{a^3+b^3}{c^3+d^3}\right)^{1/3}=\left(\frac{a^2+b^2}{c^2+d^2}\right)^{1/2}$

Sol: Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

So

L.H.S. $=\left(\frac{a^3+b^3}{c^3+d^3}\right)^{1/3}=\left(\frac{b^3k^3+b^3}{d^3k^3+d^3}\right)^{1/3}$

$=\left(\frac{b^3(k^3+1)}{d^3(k^3+1)}\right)^{1/3}$

$=\left(\frac{b^3}{d^3}\right)^{1/3}$

$=\frac{b}{d}$ ...... (1)

Now

R.H.S. $=\left(\frac{a^2+b^2}{c^2+d^2}\right)^{1/2}=\left(\frac{b^2k^2+b^2}{d^2k^2+d^2}\right)^{1/2}$

$=\left(\frac{b^2(k^2+1)}{d^2(k^2+1)}\right)^{1/2}$

$=\left(\frac{b^2}{d^2}\right)^{1/2}$

$=\frac{b}{d}$ ...... (2)

from (1) and (2)

L.H.S. = R.H.S.

(x) $\left(\frac{1}{a}-\frac{1}{b}\right)+\left(\frac{1}{d}-\frac{1}{c}\right)=\frac{(a-b)(a-c)}{abc}$

Sol: Let $\frac{a}{b}=\frac{c}{d}=k$

$\Rightarrow$ a = bk

c = dk

So

L.H.S. $=\left(\frac{1}{a}-\frac{1}{b}\right)+\left(\frac{1}{d}-\frac{1}{c}\right)=\left(\frac{1}{bk}-\frac{1}{b}\right)+\left(\frac{1}{d}-\frac{1}{dk}\right)$

$=\frac{1-k}{bk}+\frac{k-1}{dk}$

$=\frac{d(1-k)+b(k-1)}{bdk}=\frac{-d(k-1)+b(k-1)}{bdk}$

$=\frac{(k-1)(b-d)}{bdk}$ ...... (1)

Now

$$\text{R.H.S.} = \frac{(a-b)(a-c)}{abc} = \frac{(bk-b)(bk-dk)}{(bk)(b)(dk)}$$

$$= \frac{b(k-1)k(b-d)}{b^2dk^2}$$

$$= \frac{(k-1)(b-d)}{bkd} \quad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

Q.2 If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$

(where $a, b, c, d, e, f \neq 0$**) then prove that**

(i) $\frac{b^3+d^3+f^3}{a^3+c^3+e^3} = \frac{bdf}{ace}$

Sol: Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\Rightarrow$ a = bk

c = dk

e = fk

So

L.H.S.

$$= \frac{b^3+d^3+f^3}{a^3+c^3+e^3} = \frac{b^3+d^3+f^3}{b^3k^3+d^3k^3+f^3k^3}$$

$$= \frac{(b^3+d^3+f^3)}{k^3(b^3+d^3+f^3)}$$

$$= \frac{1}{k^3} \quad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = \frac{bdf}{ace} = \frac{bdf}{bk.dk.fk}$$

$$= \frac{bdf}{bdfk^3}$$

$$= \frac{1}{k^3} \quad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(ii) $\left(\frac{pa+qc+re}{pb+qd+rf}\right)^3 = \frac{ace}{bdf}$

Sol:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\Rightarrow$ a = bk

c = dk

e = fk

So

L.H.S.

$$= \left(\frac{pa+qc+re}{pb+qd+rf}\right)^3 = \left(\frac{pbk+qdk+rfk}{pb+qd+rf}\right)^3$$

$$= k^3\left(\frac{pb+qd+rf}{pb+qd+rf}\right)^3$$

$$= k^3 \quad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = \frac{ace}{bdf} = \frac{bk.dk.fk}{bdf} = \frac{bdfk^3}{bdf}$$

$$= k^3 \quad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(iii) $\frac{b^2+d^2+f^2}{ab+cd+ef} = \sqrt[3]{\frac{bdf}{ace}}$

Sol:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$\Rightarrow$ a = bk

c = dk

e = fk

So

$$\text{L.H.S.} = \frac{b^2+d^2+f^2}{ab+cd+ef} = \frac{b^2+d^2+f^2}{b^2k+d^2k+f^2k}$$

$$= \frac{(b^2+d^2+f^2)}{k(b^2+d^2+f^3)}$$

$$=\frac{1}{k} \qquad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = 3\sqrt{\frac{bdf}{ace}} = \left(\frac{bdf}{ace}\right)^{1/3}$$

$$=\left(\frac{bdf}{bdfk^3}\right)^{1/3}$$

$$=\left(\frac{1}{k^3}\right)^{1/3}$$

$$=\frac{1}{k} \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

Q.3 If x: *l* = y : m = z : n where x, y, z, *l*, m, n ≠ 0 then prove that

(i) $\frac{x^3+y^3+z^3}{l^3+m^3+n^3}=\frac{xyz}{lmn}$

Sol:

As $x:l=y:m=z:n$

$$\Rightarrow \quad \frac{x}{l}=\frac{y}{m}=\frac{z}{n}=k(say)$$

so $x = lk$

$y = mk$

$z = nk$

So,

$$\text{L.H.S.} = \frac{x^3+y^3+z^3}{l^3+m^3+n^3}=\frac{l^3k^3+m^3k^3+n^3k^3}{l^3+m^3+n^3}$$

$$=\frac{k^3(l^3+m^3+n^3)}{(l^3+m^3+n^3)}$$

$$=k^3 \qquad \ldots\ldots (1)$$

Now

$$\text{R.H.S.} = \frac{xyz}{lmn}=\frac{(lk)(mk)(nk)}{lmn}=\frac{K^3lmn}{lmn}$$

$$=k^3 \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(ii) $\frac{x^3}{l^2}+\frac{y^3}{m^2}+\frac{z^3}{n^2}=\frac{(x+y+z)^3}{(l+m+n)^2}$

Sol:

As $x:l=y:m=z:n$

$$\Rightarrow \quad \frac{x}{l}=\frac{y}{m}=\frac{z}{n}=k$$

so $x=lk$

$y=mk$

$z=nk$

Now

$$\text{L.H.S.} = \frac{x^3}{l^2}+\frac{y^3}{m^2}+\frac{z^3}{n^2}=\frac{l^3k^3}{l^2}+\frac{m^3k^3}{m^2}+\frac{n^3k^3}{n^2}$$

$$=lk^3+mk^3+nk^3$$

$$=k^3(l+m+n) \qquad \ldots\ldots (1)$$

Also

$$\text{R.H.S.} = \frac{(x+y+z)^3}{(l+m+n)^2}=\frac{(lk+mk+nk)^3}{(l+m+n)^2}$$

$$=k^3\frac{(l+m+n)^3}{(l+m+n)^2}$$

$$=k^3(l+m+n) \qquad \ldots\ldots (2)$$

from (1) and (2)

L.H.S. = R.H.S.

(iii) $\frac{l^3}{x^3}+\frac{m^3}{y^3}-\frac{n^3}{z^3}=\frac{lmn}{xyz}$

Sol:

As $x:l=y:m=z:n$

$$\Rightarrow \quad \frac{x}{l}=\frac{y}{m}=\frac{z}{n}=k$$

So $x=lk$

$y=mk$

$z=nk$

So

L.H.S. = $\frac{l^3}{x^3}+\frac{m^3}{y^3}-\frac{n^3}{z^2}=\frac{l^3}{l^3k^3}+\frac{m^3}{m^3k^3}-\frac{n^3}{n^3k^3}$

$=\frac{1}{k^3}+\frac{1}{k^3}-\frac{1}{k^3}$

$=\frac{1}{k^3}$ ...... (1)

Now

R.H.S. = $\frac{lmn}{xyz}=\frac{lmn}{lk.mk.nk}=\frac{lmn}{lmnk^3}$

$=\frac{1}{k^3}$ ...... (2)

from (1) and (2)

L.H.S. = R.H.S.

Q.4 If $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}(a,b,c,q,r\neq 0)$ **then prove that**

(i) $\frac{a+b+c}{p+q+r}=\frac{a+b-c}{p+q-r}=\frac{a-b-c}{p-q-r}$

Sol:

Let: $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}=k$

$\Rightarrow$ $a=pk$

$b=qk$

$c=rk$

L.H.S. = $\frac{a+b+c}{p+q+r}=\frac{pk+qk+rk}{p+q+r}$

$=\frac{k(p+q+r)}{(p+q+k)}$

= k ...... (1)

Now

$\frac{a+b-c}{p+q-r}=\frac{pk+qk-rk}{p+q-r}$

$=\frac{k(p+q-r)}{(p+q-r)}$

= k .... (2)

Now

$\frac{a-b-c}{p-q-r}=\frac{pk-qk-rk}{p-q-r}$

$=\frac{k(p-q-r)}{(p-q-r)}$

= k .... (3)

from (1) , (2) and (3)

$\frac{a+b+c}{p+q+r}=\frac{a+b-c}{p+q-r}=\frac{a-b-c}{p-q-r}$

(ii) $\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=\frac{qr}{bc}+\frac{rp}{ca}+\frac{pq}{ab}$

Sol: Let: $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}=k$

$\Rightarrow$ $a=pk$

$b=qk$

$c=rk$

L.H.S.

$\frac{p^2}{a^2}+\frac{q^2}{b^2}+\frac{r^2}{c^2}=\frac{p^2}{p^2k^2}+\frac{q^2}{q^2k^2}+\frac{r^2}{r^2k^2}$

$=\frac{1}{k^2}+\frac{1}{k^2}+\frac{1}{k^2}$

$=\frac{3}{k^2}$ ....(1)

So

R.H.S.

$\frac{qr}{bc}+\frac{rp}{ca}+\frac{pq}{ab}=\frac{qr}{qk.rk}+\frac{rp}{rk.pk}+\frac{pq}{pk.qk}$

$=\frac{qr}{qrk^2}+\frac{rp}{rpk^2}+\frac{pq}{pqk^2}$

$=\frac{1}{k^2}+\frac{1}{k^2}+\frac{1}{k^2}$

$=\frac{3}{k^2}$ ....(2)

from (1) and (2)

L.H.S. = R.H.S.

Exercise 3.7

Q.1 Which number is added to 5,7,9 and 12 that we get proportion in them.

Sol: Let we add the number x

Then according to given Condition

$5 + x : 7 + x :: 9 + x ; 12 + x$

$\Rightarrow \quad \frac{5+x}{7+x} : \frac{9+x}{12+x}$

or $(5 + x)(12 + x) = (7 + x)(9 + x)$

$60 + 5x + 12x + x^2 = 63 + 7x + 9x + x^2$

$17x + 60 = 16x + 63$

$17x - 16x = 63 - 60$

$x = 3$

Q.2 If 2x is added to 3,5,4 and 8, then new numbers are in proportion. Find the value of x

Sol: According to given Condition

$2x + 3 : 2x + 5 :: 2x + 4 : 2x + 8$

$\Rightarrow (2x + 3)(2x + 8) = (2x + 5)(2x + 4)$

$4x^2 + 16x + 24 = 4x^2$

$4x^2 + 16x + 6x + 24 = 4x^2 + 8x + 10x + 20$

$4x^2 + 22x + 24 = 4x^2 + 18x + 20$

$22x + 24 = 18x + 20$

$22x - 18x = 20 - 24$

$4x = -4$

$x = -1$

Q.3 Find the numbers if the ratio between them is 9 : 5 and their difference is 36.

Sol: Let the numbers be x and y

According to given condition

$\frac{x}{y} = \frac{9}{5}$

$\frac{x}{9} = \frac{y}{5} = k$

$\Rightarrow \quad x = 9k$

$y = 5k$

Also given

$x - y = 36$

$9k - 5k = 36$

$4k = 36$

$k = 9$

So $x = 9(9) = 81$

$y = 5(9) = 45$

Q.4 Find the numbers if their sum is 60 and the ratio between them is 5:7.

Sol: Let the req. numbers be x,y

According to given condition

$\frac{x}{y} = \frac{5}{7}$

$\frac{x}{5} = \frac{y}{7} = k$

$\Rightarrow \quad x = 5k$

$y = 7k$

Also given

$x + y = 60$

$5k + 7k = 60$

$12k = 60$

$k = 5$

So $x = 5k = 5(5) = 25$

$y = 7k = 7(5) = 35$

Hence req. numbers are 25,35

Q.5 The ratio among the angles of a triangle is 3 : 5 : 7. Find the measure of each angle.

Sol: let the angles be x,y,z

Here $x : y : z = 3 : 5 : 7$

or $\frac{x}{3} = \frac{y}{5} = \frac{z}{7} = k(say)$

$\Rightarrow \quad x = 3k$

$y = 5k$

$z = 7k$

Also we know

$x + y + z = 180^o$

$3k + 5k + 7k = 180^o$

$15k = 180^o$

$k = 12$

So req. of angle are

$x = 3(12) = 36^o$

$y = 5(12) = 60^o$

$z = 7(12) = 840^o$

Q.6 The ratios among the sides of a quadrilateral is 3 : 6 : 5 : 8 and its perimetre is 462 cm. Find the length of each side.

Sol: Let the sides are a,b,c,d

As given

$a : b : c : d = 3 : 6 : 5 : 8$

$\Rightarrow \quad \frac{a}{3} = \frac{b}{6} = \frac{c}{5} = \frac{d}{8} = k(say)$

so $a = 3k$

$b = 6k$

$c = 5k$

$d = 8k$

As perimeter of quadrilateral = 462

$\Rightarrow \quad a + b + c + d = 462$

$3k + 6k + 5k + 8k = 462$

$22k = 462$

$k = 21$

So req. sides are

$a = 3k = 3(21) = 63$ cm.

$b = 6k = 6(21) = 126$ cm.

$c = 5k = 5(21) = 105$ cm.

$d = 8k = 8(21) = 168$ cm.

Q.7 The ratios among the angles of a quadrilateral is 2 : 3 : 6 : 7. Find the measure of each angle.

Sol: Let the angle be a,b,c,d

As given $a:b:c:d = 2:3:6:7$

$\frac{a}{2} = \frac{b}{3} = \frac{c}{6} = \frac{d}{7} = k(say)$

$\Rightarrow a = 2k$

$b = 3k$

$c = 6k$

$d = 7k$

As we known

$a + b + c + d = 360^o$

$2k + 3k + 6k + 7k = 360^o$

$18k = 360^o$

$k = 20^o$

So req. angles are

$a = 2k = 2(20) = 40^o$

$b = 3k = 3(20) = 60^o$

$c = 6k = 6(20) = 120^o$

$d = 7k = 7(20) = 140^o$

Q.8 Shadow of a 40m high pole is 60m long. Find the height of a person if its shadow is 9m.

Sol: Let the height of person = xm

As we have

$40 : 60 :: x : 9$

$360 = 60x$

$x = \frac{360}{60}$

$x = 6$

So height of person = 6m

Q.9 The ratios between the income and expenses of Gulzaib and Shahzaib are 4 : 5 and 2 : 3 respectively. If monthly saving of each is Rs. 2500 then find the monthly income and expenses of each.

Sol: Let x and y denote monthly income of Shahzaib and Gulzaib

According to first condition

$x:y = 4:5$

As given

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x}{4} = \frac{y}{5} = k$$

$\Rightarrow$ $x = 4k, y = 5k$

According to 2nd condition

$$x - 2500 : y - 2500 = 2 : 3$$

$$\frac{x-2500}{y-2500} = \frac{2}{3}$$

Put $x = 4k$, $y = 5k$

$$\frac{4k-2500}{5k-2500} = \frac{2}{3}$$

$$3(4k - 2500) = 2(5k - 2500)$$

$$12k - 7500 = 10k - 5000$$

$$12k - 19k = 7500 - 5000$$

$$2k = 2500$$

$$k = \frac{2500}{2} = 1250$$

$\therefore$ Gulzaib's income $= x = 4k$

$= 4(1250)$

$= 5000$ Rs.

Shahzaib's income $= x = 5k$

$= 5(1250)$

$= 6250$ Rs.

Gulzaib's expenses = Income – Saving

$= 5000 - 2500$

$= 2500$ Rs.

Shahzaib's expenses = Income - Saving

$= 6250 - 2500$

$= 3750$ Rs.

Q.11 The ratio between two numbers is 8 : 5. If 2 is added to each number then the new ratio becomes 4 : 3. Find the numbers.

Sol: Let the numbers be x , y

As given

$$\frac{x}{y} = \frac{8}{5}$$

$$\Rightarrow \quad \frac{x}{8} = \frac{y}{5} = k$$

or $\left.\begin{array}{l} x = 8k \\ y = 5k \end{array}\right]$

Also given

$$\frac{x+12}{y+12} = \frac{4}{3}$$

Put $x = 8k$, $y = 5k$

$$3x + 36 = 4y + 48$$

$$\frac{8k+12}{5k+12} = \frac{4}{3}$$

$$3(8k + 12) = 4(5k + 12)$$

$$24k + 36 = 20k + 48$$

$$24k - 20k = 48 - 36$$

$$4k = 12$$

$$k = \frac{12}{4} = 3$$

$\therefore$ required numbers are, $x = 8k$

$= 8(3) = 24$

and $y = 5k$

$= 5(3) = 15$

Q.12 The ratio between two numbers is 4:5. If 11 is subtracted from each number then new ratio becomes 13:19. Find numbers.

Sol: Let required numbers are x and y

According to first condition

$$x : y = 4 : 5$$

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x}{4} = \frac{y}{5} = k$$

$\Rightarrow$ $x = 4k$, $y = 5k$

According to 2nd condition

$x - 11 : y - 11 = 13 : 19$

$$\frac{x-11}{y-11} = \frac{13}{19}$$

$$\frac{4k-11}{5k-11} = \frac{13}{19}$$

$19(4k - 11) = 13(5k - 11)$

$76k - 209 = 65k - 143$

$76k - 65k = 209 - 143$

$11k = 66$

$\Rightarrow \quad k = 6$

$\therefore$ Required numbers

$x = 4k$

$x = 4(6) = 24$

and $\quad y = 5k$

$= 5(6) = 30$

EXAMPLES

Example 1: The length and width of a rectangle is 100cm and 80cm respectively Find the ratio of length to the width.

Solution:

$$\frac{\text{Length of the rectangle}}{\text{Width of the rectangle}} = \frac{100\,\text{cm}}{80\,\text{cm}}$$

$$= \frac{4\times5\times5}{4\times4\times5}$$

$$= \frac{4\times5\times5}{4\times4\times5}$$

$$= \frac{5}{4}$$

$\therefore$ the required ratio is 5 : 4

Note:

1. The value of a ratio does not change if its numerator and denominator are both multiplied or divided by the same numbers.
2. The order of the elements of a ratio is important i.e. *a:b* and *b:a* are not equivalent generally.
3. A ratio has no unit.

Example 2: For what value of x the ratio $2 + x : x + 12$ and the ratio $2 : 3$ equal.

Solution:

let $\dfrac{2+x}{x+12} = \dfrac{2}{3}$. Then

$3(2 + x) = 2(x + 12)$

or $6 + 3x = 2x + 24$

or $3x - 2x = 2x + 24$

or $x = 18$

Example 3: If $x : y = 4 : 5$, then find $3x + 2y : 2x + 4y$

Solution: Here $x : y = 4 : 5$

or $\dfrac{x}{y} = \dfrac{4}{5}$

Now

$$3x+2y : 2x+4y = \frac{3x+2y}{2x+4y} = \frac{3\left(\frac{x}{y}\right)+2\left(\frac{y}{y}\right)}{3\left(\frac{x}{y}\right)+4\left(\frac{x}{y}\right)}$$

$$= \frac{3\left(\frac{x}{y}\right)+2}{2\left(\frac{x}{y}\right)+4} = \frac{3\left(\frac{x}{y}\right)+2}{2\left(\frac{x}{y}\right)+4} \left(\because \frac{x}{y} = \frac{4}{5}\right)$$

$$= \frac{\frac{12}{5}+2}{\frac{8}{5}+2} = \frac{\frac{12+10}{5}}{\frac{8+20}{5}} = \frac{\frac{22}{5}}{\frac{28}{5}}$$

$$= \frac{22}{28} = \frac{11}{14}$$

Thus $3x + 2y : 2y + 4y = 11 : 14$

Example 4: The ratio of two numbers is $2 : 3$ If 8 is added to both the numbers, then the new ratio becomes $4 : 5$ Find the numbers.

Solution: Since ratio of two numbers is $2 : 3$

Let the number be $2x$ and $3x$.

Then according to the given condition,

$$\frac{2x+8}{3x+8} = \frac{4}{5}$$

or $5(2x + 8) = 4(3x + 8)$

(By cross multiplication)

or $10x + 40 = 12x + 32$

or $10x - 12x = 32 - 40$

or $-2x = -8$

or $x = 4$

$\therefore 2x = 2(4) = 8$

and $3x = 3(4) = 12$

Thus the required numbers are 8 and 12.

Example 5: Find the value of x if $5 : 2x : : 3 : 2x - 4$

Solution: $5 : 2x : : 3 : 2x - 4$

$\Rightarrow 5(2x - 4) = 3(2x)$ (Product of extremes = Product of means

or $10x - 20 = 6x$

or $10x - 6x = 20$

or $4x = 20$

or $x = 5$

Example 6: If we subtract the same number from each of 12, 14, 20 and 30, then we get a proportion in them. Find that number.

Solution: Let x be the required number

According to the given condition.

$12 - x : 14 - x : : 20 - x : 30 - x$

We know that, in a proportion

Product of the means = Product of the extremes

$\therefore (14 - x)(20 - x) = (12 - x)(30 - x)$

or $280 - 34x + x^2 = 360 - 42x + x^2$

or $-34x + 42x = 360 - 280$

or $8x = 80$

or $x = 10$

Thus, 10 is the required number.

Example 7: x and y vary directly and $y = 4$ for $x = 12$ If $x = 18$, then find y

Solution: As x and y vary directly so,

$$\frac{y}{x} = k \qquad \text{(i)}$$

Putting the value of $y = 4$ and $x = 12$ in the equation (i), we have

$$\frac{4}{12} = k \quad or \quad k = \frac{1}{3}$$

Now by putting $k = \frac{1}{3}$ and $x = 18$ in the equation (i), we get

$$\frac{y}{18} = \frac{1}{3}$$

or $y = 6$

Example 8: a and b^3 vary directly, and, $a = 27$ when $b = 2$

Find the value of b when $a = 8$,

Solution: Since a and b^3 vary directly, so

$$a \propto b^3$$

$$\Rightarrow a = kb^3 \qquad \text{(i)}$$

$$\Rightarrow \frac{a}{b^3} = k$$

Putting the given value of a and b in the equation (i), we have

$27 = k(2)^3$

or $27 = 8k$

or $\frac{27}{8} = k$

Putting $k = \frac{27}{8}$ and $a = 8$ in the equation (i), we get

$$a = \frac{27}{8}b^3$$

$$8 = \frac{27}{8}b^3$$

or $\frac{64}{27} = b^3$

or $b^3 = \left(\frac{4}{3}\right)^3$

or $b = \frac{4}{3}$

Example 9:

$A = \frac{3168}{7}cm^2$ and $r = 12cm$ and $A \propto r^2$

If $A = \frac{1408}{7}cm^2$, then find r.

Solution: Since $A \propto r^2$ Then

$A = kr^2$ (i)

Now putting $r = 12cm$ and $A = \frac{3168}{7}cm^2$ in equation (i), we get

$$\frac{3168}{7} = k(12)^2$$

or $\frac{3168}{7} = 144k$

or $k = \frac{3168}{7 \times 144}$

or $k = \frac{22}{7}$

putting $k = \frac{22}{7}$ and $A = \frac{1408}{7}cm^2$ in equation (i), we get

$\frac{1408}{7} = \frac{22}{7}r^2$ (putting the value of a and k)

or $1408 = 22r^2$

or $64 = r^2$

or $r = \pm 8$

Since radius is always a positive quantity.

$\therefore$ $r = 8$

Example 10: $v \propto h^3$ and $v = 40$ when $h = 2$. If $v = 320$, then find h.

Solution: Since $v \propto h^3$

$\therefore$ $v = kh^3$ (i)

Now putting the value of v and h in equation (i), we have

$$40 = k(2)^3$$

or $8k = 40$

or $k = 5$

Now putting $k = 5$ and $v = 320$ in equation (i), we have

$$320 = 5h^3$$

$$\frac{320}{5} = h^3$$

or $h^3 = 64$

or $h = 4$

Thus $h = 4$ for $v = 320$

Example 11: x and y vary inversely and $y = 12$ for $x = 5$ Find y, if $x = 15$

Solution: As x and y vary inversely, so we have $y \propto \frac{1}{x} \Rightarrow y = \frac{k}{x}$

$yx = k$ (i)

or $12(5) = k$ (by substituting values)

or $k = 60$

Now putting $x = 15$ and $k = 60$ in the equation (i), we have

$y(15) = 60$

$\therefore$ $y = 4$

Example 12:

$y \propto \frac{1}{x^2}$ and $y = 9$ for $x = 2$. Find y for $x = 4$

Solution:

Given that $y \propto \frac{1}{x^2}$

$\Rightarrow \quad y = \frac{k}{x^2}$

or $\qquad yx^2 = k \qquad$ (i)

putting the given values of x and y in equation (i), we have

$9(2)^2 = k$

$k = 9(4) = 36$

putting $k = 36$ and $x = 4$ in the equation (i), we have

$y(4)^2 = 36$

or $\quad 16y = 36$

$y = \frac{9}{4}$

Example 13: Find the fourth proportional of $m^2 - n^2$, m and $m - n$

Solution: $m^2 - n^2 : m :: m - n : x$

or $\quad x(m^2 - n^2) = m(m - n)$

or $\quad x(m - n)(m + n) = m(m - n)$

or $\quad x = \frac{m\cancel{(m-n)}}{(m+n)\cancel{(m-n)}}$

$= \frac{m}{m+n}$

$\therefore$ the fourth proportional is $\frac{m}{m+n}$

Example 14: Find the third proportional of $a^2 + b^2$ and $a^4 - b^4$

Solution: Let x be the third proportional. Then

$a^2 + b^2 : a^4 - b^4 :: a^4 - b^4 : x$

or $\quad x = \frac{(a^4 - b^4)(a^4 - b^4)}{(a^2 + b^2)}$

$x = \frac{(a^2+b^2)(a^2-b^2)(a^4-b^4)}{(a^2+b^2)}$

or $\quad x = (a^2 - b^2)(a^4 - b^4)$

or $\quad x = (a^2 - b^2)(a^2 - b^2)(a^2 + b^2)$

$\therefore \quad x = (a^2 - b^2)^2 (a^2 + b^2)$

Example 15: Find the mean proportional of $x^4 y^4$ and z^2.

Solution: Let m be the mean proportional. Then

$x^4y^4 : m :: m : z^2$

or $\quad m^2 = x^4 y^4 z^2$

or $\quad m = \pm\sqrt{x^4 y^4 z^2}$

$\therefore \quad m = \pm x^2 y^2 z$

Example 16: If 16, a and 4 are in continued proportion, then find a

Solution: Since there is a continued proportion

$\therefore \quad 16 : a :: a : 4$

or $\quad a^2 = 64$

$\therefore \quad a = \pm 8$

Example 17: If $2x : 3y - 1 : 2m : n + 1$, then $3y - 1 : 2x = n + 1 : 2m$

Solution: Let $2x : 3y - 1 = 2m : n + 1$ Then

$\frac{2x}{3y-1} = \frac{2m}{n+1}$

or $\quad (2x)(n + 1) = (2m)(3y - 1)$

or $\quad (2m)(3y - 1) = (2x)(n + 1)$

or $\quad \frac{\cancel{(2m)}(3y-1)}{\cancel{(2m)}(2x)} = \frac{\cancel{(2x)}(n+1)}{(2m)\cancel{(2x)}}$

or $\quad \frac{3y-1}{2x} = \frac{n+1}{2m}$

or $\quad 3y - 1 : 2x = n + 1 : 2m$

Therefore, if $2x : 3y - 1 = 2m : n + 1$, then $3y - 1 : 2x = n + 1 : 2m$

Example 18: If $3m : 2n = x - 1 : y + 1$, then

$3m : 2n = x - 1 : y + 1$

Solution: Let $3m : 2n = x - 1 : y + 1$

Then

$$\frac{3m}{2n} = \frac{x-1}{y+1}$$

or $(3m)(y+1) = (2n)(x-1)$

or $$\frac{(3m)(y+1)}{(x-1)(y+1)} = \frac{(2n)(x-1)}{(x-1)(y+1)}$$

[dividing by (x - 1)(y + 1)]

or $$\frac{3m}{x-1} = \frac{2n}{y+1}$$

or $3m : x - 1 : 2n : y + 1$

Therefore, if $3m : 2n = x - 1 : y + 1$, then $3x : x - 1 = 2n : y + 1$

Therefore, if a : b = c : d, then a : a + b = c : c + d

Example 19: If m : n + 1 = x - 1 : y then: $m + n + 1 : n + 1 = x + y - 1 : y$

Solution: Let $m : n + 1 = x + y - 1 : y$

$$\frac{m}{n+1} = \frac{x-1}{y}$$

or $\frac{m}{n+1} + 1 = \frac{x-1}{y} + 1$ (adding 1 on both the sides)

or $$\frac{m+n+1}{n+1} = \frac{x-1+y}{y}$$

or $$\frac{m+n+1}{n+1} = \frac{x+y-1}{y}$$

or $m+n+1 : n+1 = x+y-1 : y$

Example 20: If $3m : 2n = 2x + 1 : 2y - 1$, then $3m - 2n : 2n = 2x - 2y + 2 : 2y - 1$

Solution: Let $3m : 2n = 2x + 1 : 2y - 1$,

Then

$$\frac{3m}{2n} = \frac{2x+1}{2y-1}$$

or $$\frac{3m}{2n} - 1 = \frac{2x+1}{2y-1} - 1$$

(subtracting 1 from both the sides)

or $$\frac{3m-2n}{2n} = \frac{2x+1-2y+1}{2y-1}$$

or $$\frac{3m-2n}{2n} = \frac{2x-2y+2}{2y-1}$$

or $$\frac{3m-2n}{2n} = \frac{2x-2y+2}{2y-1}$$

or $3m - 2n : 2n = 2x - 2y + 2 : 2y - 1$

Therefore, If $3m : 2n = 2x + 1 : 2y - 1$, then $3m - 2n : 2n = 2x - 2y + 2 : 2y - 1$

Example 21: If $a : b = c : d$, then prove that 4a+5b : 4a–5b = 4c+5d : 4c-5d

Solution:

Let $a : b = c : d$

or $$\frac{a}{b} = \frac{c}{d}$$

or $\frac{4a}{5b} = \frac{4c}{5d}$ (multiplying both the sides with $\frac{4}{5}$)

or $\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$ (by componendo - dividendo theroem)

or $4a + 5b : 4a - 5b = 4c + 5d : 4c - 5d$

Therefore, $4a + 5b : 4a - 5b = 4c + 5b : 4c - 5d$ if $a : b = c : d$

Example 22:

If $\frac{2a+3b}{2a-3b} = \frac{2c+3d}{2c-3d}$, then prove that a : b = c : d

Solution:

Let $$\frac{2a+3b}{2a-3b} = \frac{2c+3d}{2c-3d}$$

$\therefore \dfrac{(2a+3b)+(2a-3b)}{(2a+3b)-(2a-3b)} = \dfrac{(2c+3d)+(2c-3d)}{(2c-3d)-(2c-3d)}$

by componendo- dividend

$$\frac{2a+3b+2a-3b}{2a+3b-2a-3b} = \frac{2c+3d+2c-3d}{2c-3d-2c-3d}$$

or $\dfrac{4a}{6b} = \dfrac{4c}{6d}$

$\dfrac{4a}{6b}\times\dfrac{6}{4} = \dfrac{4c}{6d}\times\dfrac{6}{4}$ Multiplying both sides by $\dfrac{6}{4}$

or $\dfrac{a}{b} = \dfrac{c}{d}$

or $a : b = c : d$

Example 23:

If $m = \dfrac{4ab}{a+b}$, then find the value of $\dfrac{m+2a}{m-2a} - \dfrac{m+2b}{m-2b}$ by using componendo-dividendo property.

Solution:

Let $m = \dfrac{4ab}{a+b}$ (i)

$\therefore \dfrac{m}{2a} = \dfrac{2b}{a+b}$

or $\dfrac{m+2a}{m-2a} = \dfrac{2b+a+b}{2b-(a+b)}$ (by componendo-dividendo theorem)

or $\dfrac{m+2a}{m-2a} = \dfrac{2b+a+b}{b-a}$ (ii)

Again from equation (i), we get

$\dfrac{m}{2b} = \dfrac{2b}{a+b}$

or $\dfrac{m+2b}{m-2b} = \dfrac{2b+(a+b)}{2a-(b+a)}$

(Componendo-dividendo theorem)

or $\dfrac{m+2a}{m-2a} = \dfrac{3a+b}{a-b}$ (iii)

From equation (ii) and (iii), we get

$$\frac{m+2a}{m-2a} - \frac{m+2a}{m-2a} = \frac{a+3b}{b-a} - \frac{3a+b}{a-b}$$

$$= \frac{-a-3b}{a-b} - \frac{3a+b}{a-b}$$

$$= \frac{-a-3b-3a-b}{a-b}$$

$$= \frac{-4a-4b}{a-b}$$

$$= \frac{-4(a+b)}{a-b}$$

Example 24:

Solve the following equation by using componendo-dividendo property

$$\frac{(x+7)^2+(x-5)^2}{(x+7)^2-(x-5)^2} = \frac{5}{4}$$

Solution: By componendo - dividendo theorem, we have

$$\frac{(x+7)^2+(x-5)^2+(x+7)^2+(x-5)^2}{(x+7)^2+(x-5)^2-(x+7)^2+(x-5)^2} = \frac{5=4}{5-4}$$

or $\dfrac{2(x+7)^2}{2(x-5)^2} = \dfrac{9}{1}$

or $\left(\dfrac{x+7}{x-5}\right)^2 = (\pm 3)^2$

or $\dfrac{x+7}{x-5} = \pm 3$

$\Rightarrow \dfrac{x+7}{x-5} = 3$ *or* $\dfrac{x+7}{x-5} = -3$

$\Rightarrow$ *x + 7 = 3(x - 5) or x + 7 = -3 (x - 5)*

$\Rightarrow$ *x + 7 = 3x – 15 orx + 7 = -3x - 15*

$\Rightarrow$ *7 + 15 = 3x - x or x + 3x = 15 - 7*

$\Rightarrow$ 22 = 2x or 4x = 8

$\Rightarrow$ x = 11 or x = 2

Therefore, solution set = {2, 11}

Example 25:

If $\frac{a}{b}=\frac{c}{d}$, then prove that

$$\frac{3ac+4bd}{3ac-4bd}=\frac{3a^2+4b^2}{3a^2-4b^2}$$

Solution:

Let $\frac{a}{b}=\frac{c}{d}=k$. Then $a=bk$ and $c=dk$

Consider

$$\frac{3ac+4bd}{3ac-4bd}=\frac{3(bk)(dk)+4bd}{3(bk)(dk)-4bd}$$

$\therefore$ (*putting* $a=bk$ *and* $c=dk$)

$$=\frac{bd\,\cancel{(3k^2+4)}}{bd\,\cancel{(3k^2-4)}} \qquad \text{(i)}$$

Now consider

$$\frac{3a^2+4b^2}{3a^2-4b^2}=\frac{3(bk)^2+4b^2}{3(bk)^2-4b^2}$$

(putting $a=bk$ and $c=dk$)

$$=\frac{3b^2k^2+4b^2}{3b^2k^2-4b^2}$$

$$=\frac{b^2\,\cancel{(3k^2+4)}}{b^2\,\cancel{(3k^2-4)}}$$

$$\therefore \quad \frac{3a^2+4b^2}{3a^2-4b^2}=\frac{3k^2+4}{3k^2-4} \qquad \text{(ii)}$$

From equations (i) and (ii), we have

$$\frac{3ac+4bd}{3ac-4bd}=\frac{3a^2+4b^2}{3a^2-4b^2}$$

Example 26:

If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then prove that

$$\frac{x^3+y^3+z^3}{a^3+b^3+c^3}=\frac{xyz}{abc}$$

Solution:

$\frac{x}{a}=\frac{y}{b}=\frac{z}{c}=k$. Then

$$\frac{x}{a}=k \quad \Rightarrow \quad x=ak$$

Similarly, $y=bk$ and $z=ck$

Consider

$$\frac{x^3+y^3+z^3}{a^3+b^3+c^3}=\frac{(ak)^3+(bk)^3+(ck)^3}{a^3+b^3+c^3}$$

(Putting the value of x,y and z)

$$=\frac{k^3(a^3+b^3+c^3)}{a^3+b^3+c^3}$$

$$\therefore \quad \frac{x^3+y^3+z^3}{a^3+b^3+c^3}=k^3 \qquad \text{(i)}$$

Now consider

$\frac{xyz}{abc}=\frac{(ak)\times(bk)\times(ck)}{abc}$ (Putting the value of x, y and z)

$$=\frac{abck^3}{abc}$$

$$\therefore \quad \frac{xyz}{abc}=k^3 \qquad \text{(ii)}$$

From equations (i) and (ii), we have

$$\frac{x^3+y^3+z^3}{a^3+b^3+c^3}=\frac{xyz}{abc}, \ \text{if} \ \frac{x}{a}=\frac{y}{b}=\frac{z}{c}$$

Example 27:

If $\frac{u}{v}=\frac{w}{x}=\frac{y}{z}$, then prove that

$$\left(\frac{u^3v+w^3x+y^3z}{u^2v^2+w^2x^2+y^2z^2}\right)^{\frac{3}{2}}=\sqrt{\frac{uwy}{vxz}}$$

Solution:

Let $\frac{u}{v}=\frac{w}{x}=\frac{y}{z}=k$, then

$$\frac{u}{v}=k \Rightarrow u \Rightarrow vk$$

$w=xk$ and $y=zk$

Putting these values in both the sides of the given equation (i), we have

$$\text{L.H.S} = \left(\frac{u^3v + w^3x + y^3z}{u^2v^2 + w^2x^2 + y^2z^2}\right)^{\frac{3}{2}}$$

$$= \left(\frac{k^3u^3v + k^3x^3.x^3 + k^3z^3z}{k^2v^2.v^2 + k^2x^2x^2 + k^2z^2.z^2}\right)^{\frac{3}{2}}$$

$$= \left(\frac{k^3(v^4 + x^4 + z^4)}{k^2(v^4 + x^4 + z^4)}\right)^{\frac{3}{2}}$$

$$= k^{\frac{3}{2}}$$

R.H.S

$$\sqrt{\frac{\text{uwy}}{\text{v x z}}} = \sqrt{\frac{\text{kv.kx.kz}}{\text{v x z}}}$$

$$= \sqrt{\frac{\text{k}^3\text{ v x z}}{\text{v x z}}} = \sqrt{\text{k}^3} = \text{k}^{3/2}$$

Thus L.H.R = R.H.S

Example 28:

If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, then prove that $\frac{x-y}{a-b} = \frac{y-z}{b-c} = \frac{z-c}{c-a}$

Solution:

Let $\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$. Then

$x = ak$, $y = bk$ and $z = ck$

Consider

$$\frac{x-y}{a-b} = \frac{ak-bk}{a-b} \qquad \text{(Putting Values)}$$

$$= \frac{k(a-b)}{a-b}$$

or $\frac{x-y}{a-b} = k$ .... (i)

Now consider $\frac{y-z}{b-c} = \frac{bk-ck}{b-c}$

or $\frac{y-z}{b-c} = \frac{k(b-c)}{b-c} = k$ ....(ii)

and $\frac{z-x}{c-a} = \frac{kc-ka}{c-a}$

or $\frac{z-x}{c-a} = \frac{k(c-a)}{c-a} = \text{k}$ ....(iii)

From equations (i), (ii) and (iii) we have

$\frac{x-y}{a-b} = \frac{y-z}{b-c} = \frac{z-x}{c-a}$, if $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

Example 29: The sum of two numbers is 144. The ratio between them is 5 : 7 Find the numbers.

Solution: Suppose that the numbers are x and y.

According to given condition, we have

$x + y = 144$ (i)

and $x : y = 5 : 7$

or $\frac{x}{y} = \frac{5}{7}$

Let $\frac{x}{y} = \frac{5}{7} = k$ Then

$x = 5k$, $y = 7k$

Putting the value of x and y in equation (i), we get

$5k + 7k = 144$

or $12k = 144$

or $k = 12$

Therefore, $x = 5k = 5(12) = 60$

And $y = 7k = 7(12) = 84$

Thus, the required numbers are 60 and 84.

Example 30: The ratio among the sides of a triangle are 3, 4 and 5 Its perimeter is 156m. Find the length of each side.

Solution: Since the lengths of the sides of the triangle are in the ratio 3 : 4 : 5

Let $3k$, $4k$ and $5k$ be lengths of the sides of the triangle.

therefore, $3k + 4k + 5k = 156$

or $12k = 156$

or $k = 13$

Therefore, lengths of sides are as follows:

Length of one side $= 3k = 3(13) = 39$ m

Length of second side $= 4k = 4(13) = 52$ m

Length of third side $= 5k = 5(13) = 65$ m

$\therefore$ Required sides of the triangle are $39m$, $52m$ and $65m$

Example 31: The ratios between two numbers is $5:6$ If 4 is added to each of them, the new ratios becomes $7:8$.

Solution: Let the numbers be x and y

According to the given condition, we have

$x : y = 5 : 6$

or $x : 5 = y : 6$

or $\frac{x}{5} = \frac{y}{6} = k$

or $x = 5k$, $y = 6k$

Now $x+4 : y+4 = 7 : 8$

(according to second condition)

or $\frac{x+4}{y+4} = \frac{7}{8}$ (i)

Putting the value of x and y in equation (i), we have

$\frac{5k+4}{6k+4} = \frac{7}{8}$

or $8(5k+4) = 7(6k+4)$

or $50k + 32 = 42k + 28$

or $-2k = -4$

or $k = 2$

$\therefore$ the first number = x = 5k = 5(2) = 10

and the second number = y = 6k = 6(2) = 12

$\therefore$ the required numbers are 10 and 12

Invertendo Theorem

If $a : b = c : d$, then $b : a = d : c$

Proof: Let $a : b = c : d$ Then

$\frac{a}{b} = \frac{c}{d}$

or ad = cd

or bc = ad

or $\frac{\cancel{bc}}{\cancel{ac}} = \frac{\cancel{ad}}{\cancel{ac}}$ (dividing by ac)

or $\frac{b}{a} = \frac{d}{c}$

or b : a = d : c

Therefore, if a : b = c : d , then b : a = d : c

Alternendo Theorem

If $a : b = c : d$, then $a : c = b : d$

Proof: Let $a : b = c : d$ then

$\frac{a}{b} = \frac{c}{d}$

or $ad = cd$

or $\frac{ad}{cd} = \frac{bc}{cd}$ (dividing by cd)

or $\frac{a}{c} = \frac{b}{d}$

or $a : c = c : d$, then $a : c = b : d$

Objective

Q. 1. Four answers of each item are given from which only one is true. Tick the correct answer.

1. A relation between two quantities of the same kind and having the same units is called ____________.
(a) ratio (b) proportion
(c) direct variation
(d) inverse variation.

2. The ratio between two quantities a and b is denoted as.
(a) b : a (b) a : b
(c) $\frac{b}{a}$ (d) None of these

3. The ________ of the elements of ratio is important.
(a) order (b) unit
(c) kind (d) None of these

4. The length and width of a rectangle is 100 cm and 80cm respectively the ratio between the two quantities are __________.
(a) 5 : 4 (b) 3 : 4
(c) 5 : 3 (d) 3 : 5

5. If (2 + x) : (x +12) and ratio 2 : 3 are equal then x = _________.
(a) 16 (b) 18
(c) 19 (d) 15

6. The statement of equality between two ratios a : b and c : d is called a __________ for four quantities.
(a) ratio (b) proportion
(c) direct variation
(d) inverse variation

7. If 5 : 2x :: 3 : (2x − 4) then x = ___________.
(a) 6 (b) 4
(c) 5 (d) 2

8. If (2x −1) : 3 :: 5x : 10 then x = ___________.
(a) 2 (b) 4
(c) 6 (d) 5

9. If $(a^2 - b^2)$: (a − b) :: (a + b) : ? then ? = ________
(a) 2 (b) 1
(c) 3 (d) a − b

10. If a: b = 3 : 4 then 5a + 4b : 6a + 9b = ______________.
(a) 31 : 54 (b) 31 : 55
(c) 31 : 56 (d) 31 : 57

11. Variation are of __________ types.
(a) 2 (b) 3
(c) 4 (d) 6

12. If a and b^3 vary directly, a = 27 when b =2, if a =8 then b = _____.
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$
(c) 3 (d) 4

13. If $v \propto h^3$ and v = 40 when h =2, if v=320 then h = __________.
(a) 4 (b) 3
(c) 5 (d) 7

14. x and y varies inversely and y = 12 for x = 5 if x = 15 then y = _________
(a) 3 (b) 4
(c) 2 (d) 1

15. If $s \propto t$ then.
(a) s = kt (b) st = k
(c) st = 1 (d) s = t

16. If $p^2 \propto \frac{1}{q^3}$ then.
(a) $p^2 = \frac{k}{q}$ (b) $p^2 = \frac{k}{q^3}$
(c) $p^2 = kq$ (d) $p^2 = kq^3$

17. If 5 : 8 = 5 : x then x is
(a) 5 (b) 25
(c) 40 (d) 8

18. If $\frac{a}{b} = \frac{b}{c} = k$ then

(a) $a = ck^2$ (b) $a = bk^2$
(c) $a = c^2k$ (d) $a = b^2k$

19. If $\frac{a}{b} = \frac{b}{c}$ then.

(a) $b = ac$ (b) $c^2 = ab$
(c) $a^2 = bc$ (d) $b^2 = ac$

20. The fourth proportional of $3a^2b^2$, $5ab^2$ and $9ab$ is ________.

(a) 15ab (b) 15b
(c) 15a (d) None of these

21. The third proportional of $(a-2)^2$, (a^2-a-2) is ________.

(a) $(a+1)^2$ (b) $(a-1)^2$
(c) $a+1$ (d) $a-1$

22. The mean proportional of $(x - 2)(x+3)(x^2+x-6)$ is ______.

(a) $x+2$ (b) x^2+x-6
(c) $x+3$ (d) None of these

23. The third proportional of a^2 and b is.

(a) ab (b) $\frac{a}{b}$
(c) $\frac{a^2}{b^2}$ (d) $\frac{b^2}{a^2}$

24. If a:b=c:d then alternendo property is

(a) $\frac{a}{c} = \frac{b}{d}$ (b) $\frac{a}{b} = \frac{c}{d}$
(c) $\frac{a+b}{b} = \frac{c+d}{d}$ (d) $\frac{a-b}{d} = \frac{c-d}{d}$

25. If $\frac{a}{b} = \frac{c}{d}$ then invertendo property is: $\frac{a}{b} = \frac{c}{d}$

(a) $\frac{a}{a+b} = \frac{c}{c+d}$ (b) $\frac{a-b}{b} = \frac{c-d}{d}$
(c) $\frac{a}{c} = \frac{b}{d}$ (d) $\frac{b}{a} = \frac{c}{d}$

26. Example of direct variation in our daily life is:

(a) work and days
(b) Number of workers and number of days
(c) Food and men
(d) Volume and Pressure.

27. Example of inverse variation in our daily life is:

(a) Current and resistance
(b) Temperature and volume
(c) Weight and mass of the body
(d) Circumference and radius

Answers

1.	a	**2.**	b	**3.**	a	**4.**	a	**5.**	b	**6.**	b	**7.**	c
8.	a	**9.**	b	**10.**	a	**11.**	a	**12.**	b	**13.**	a	**14.**	b
15.	a	**16.**	b	**17.**	d	**18.**	a	**19.**	d	**20.**	b	**21.**	a
22.	b	**23.**	d	**24.**	a	**25.**	d	**26.**	a	**27.**	a		